Chapter 9

Chapter 9 Opener

Try It Yourself (p. 453)

1. $\sqrt{81} = \sqrt{9^2} = 9$

2. $-\sqrt{169} = -\sqrt{13^2} = -13$

3. $\pm\sqrt{\frac{9}{25}} = \pm\sqrt{\frac{3^2}{5^2}} = \pm\frac{3}{5}$

4. $-\sqrt{6.25} = -\sqrt{2.5^2} = -2.5$

5. $\sqrt{54} = \sqrt{9 \bullet 6} = \sqrt{9} \bullet \sqrt{6} = 3\sqrt{6}$

6. $\sqrt{80} = \sqrt{16 \bullet 5} = \sqrt{16} \bullet \sqrt{5} = 4\sqrt{5}$

7. $\sqrt{200} = \sqrt{100 \bullet 2} = \sqrt{100} \bullet \sqrt{2} = 10\sqrt{2}$

8. $x^2 + 10x + 25 = x^2 + 2(x)(5) + 5^2 = (x + 5)^2$

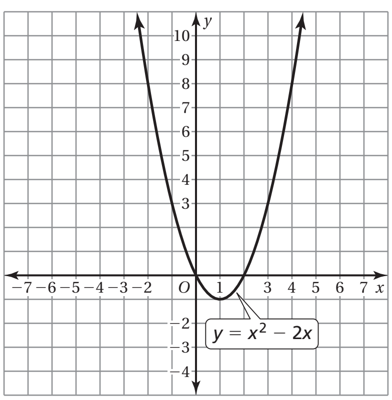
9. $m^2 - 20m + 100 = m^2 + 2(m)(-10) + (-10)^2$
$= (m - 10)^2$

10. $p^2 + 12p + 36 = p^2 + 2(p)(6) + 6^2 = (p + 6)^2$

Section 9.1

9.1 Activity (pp. 454–455)

1. a.

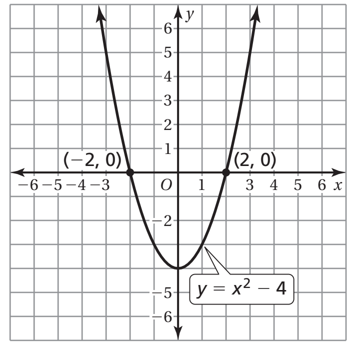


b. The x-intercepts are points where the graph crosses the x-axis. In the graph of $y = x^2 - 2x$, there are 2 x-intercepts. They are $(0, 0)$ and $(2, 0)$.

c. A solution of an equation in x is an x-value that makes the equation true. The equation $x^2 - 2x = 0$ has two solutions. They are $x = 0$ and $x = 2$.

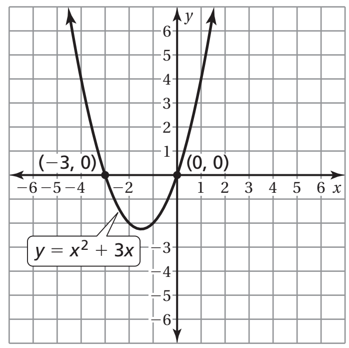
d. You can verify that the x-values found in part (c) are solutions of $x^2 - 2x = 0$ by substituting the x-values into the left side of the equation and making sure it equals zero.

2. a. $x^2 - 4 = 0$



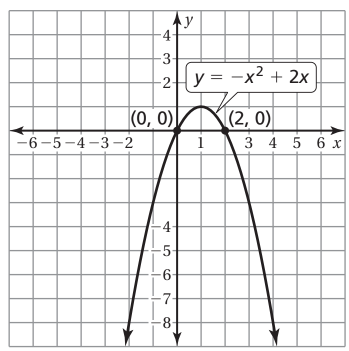
The solutions are $x = -2$ and $x = 2$.

b. $x^2 + 3x = 0$



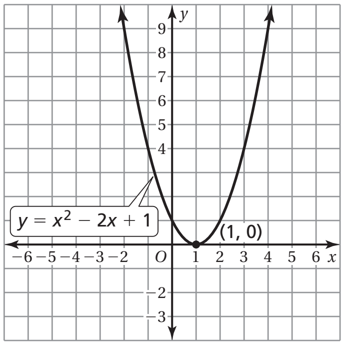
The solutions are $x = -3$ and $x = 0$.

c. $-x^2 + 2x = 0$



The solutions are $x = 0$ and $x = 2$.

d. $x^2 - 2x + 1 = 0$



The solution is $x = 1$.

3. You can get the equation equal to 0, set it equal to y, and then graph the resulting equation. The solutions of a quadratic equation in one variable are the x-intercepts of the graph.

4. You can substitute the solutions for the variable and make sure the equation is true.

Activity 2a. $x = -2$ and $x = 2$

$x = -2$:

$$x^2 - 4 = 0$$
$$(-2)^2 - 4 \stackrel{?}{=} 0$$
$$4 - 4 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

$x = 2$:

$$x^2 - 4 = 0$$
$$2^2 - 4 \stackrel{?}{=} 0$$
$$4 - 4 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Activity 2b. $x = -3$ and $x = 0$

$x = -3$:

$$x^2 + 3x = 0$$
$$(-3)^2 + 3(-3) \stackrel{?}{=} 0$$
$$9 - 9 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

$x = 0$:

$$x^2 + 3x = 0$$
$$0^2 + 3(0) \stackrel{?}{=} 0$$
$$0 + 0 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Activity 2c. $x = 0$ and $x = 2$

$x = 0$:

$$-x^2 + 2x = 0$$
$$-0^2 + 2(0) \stackrel{?}{=} 0$$
$$-0 + 0 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

$x = 2$:

$$-x^2 + 2x = 0$$
$$-2^2 + 2(2) \stackrel{?}{=} 0$$
$$-4 + 4 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

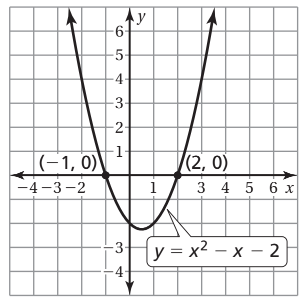
Activity 2d. $x = 1$

$x = 1$:

$$x^2 - 2x + 1 = 0$$
$$1^2 - 2(1) + 1 \stackrel{?}{=} 0$$
$$1 - 2 + 1 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

9.1 On Your Own (pp. 456–458)

1. $x^2 - x - 2 = 0$



So, the solutions are $x = -1$ and $x = 2$.

Check:

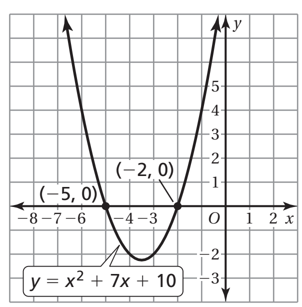
$x = -1$:

$$x^2 - x - 2 = 0$$
$$(-1)^2 - (-1) - 2 \stackrel{?}{=} 0$$
$$1 + 1 - 2 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

$x = 2$:

$$x^2 - x - 2 = 0$$
$$2^2 - 2 - 2 \stackrel{?}{=} 0$$
$$4 - 2 - 2 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

2. $x^2 + 7x + 10 = 0$



So, the solutions are $x = -5$ and $x = -2$.

Check:

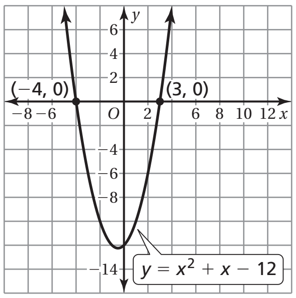
$x = -5$:

$$x^2 + 7x + 10 = 0$$
$$(-5)^2 + 7(-5) + 10 \stackrel{?}{=} 0$$
$$25 + 7(-5) + 10 \stackrel{?}{=} 0$$
$$25 - 35 + 10 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

$x = -2$:

$$\begin{aligned} x^2 + 7x + 10 &= 0 \\ (-2)^2 + 7(-2) + 10 &\stackrel{?}{=} 0 \\ 4 + 7(-2) + 10 &\stackrel{?}{=} 0 \\ 4 - 14 + 10 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

3. $x^2 + x = 12$



So, the solutions are $x = -4$ and $x = 3$.

Check:

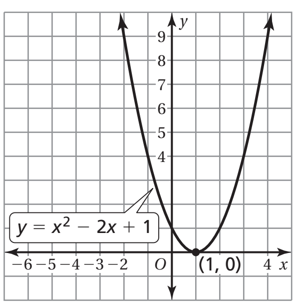
$x = -4$:

$$\begin{aligned} x^2 + x &= 12 \\ (-4)^2 + (-4) &\stackrel{?}{=} 12 \\ 16 - 4 &\stackrel{?}{=} 12 \\ 12 &= 12 \checkmark \end{aligned}$$

$x = 3$:

$$\begin{aligned} x^2 + x &= 12 \\ 3^2 + 3 &\stackrel{?}{=} 12 \\ 9 + 3 &\stackrel{?}{=} 12 \\ 12 &= 12 \checkmark \end{aligned}$$

4. $x^2 + 1 = 2x$



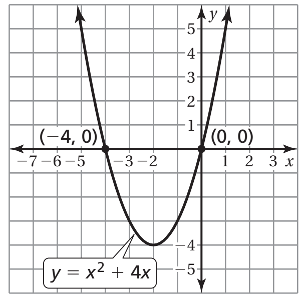
So, the solution is $x = 1$.

Check:

$x = 1$:

$$\begin{aligned} x^2 + 1 &= 2x \\ 1^2 + 1 &\stackrel{?}{=} 2(1) \\ 1 + 1 &\stackrel{?}{=} 2 \\ 2 &= 2 \checkmark \end{aligned}$$

5. $x^2 + 4x = 0$



So, the solutions are $x = -4$ and $x = 0$.

Check:

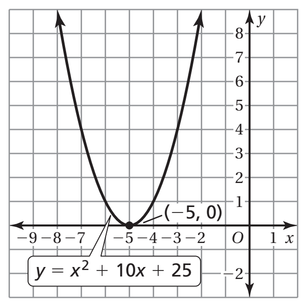
$x = -4$:

$$\begin{aligned} x^2 + 4x &= 0 \\ (-4)^2 + 4(-4) &\stackrel{?}{=} 0 \\ 16 + 4(-4) &\stackrel{?}{=} 0 \\ 16 - 16 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

$x = 0$:

$$\begin{aligned} x^2 + 4x &= 0 \\ 0^2 + 4(0) &\stackrel{?}{=} 0 \\ 0 + 4(0) &\stackrel{?}{=} 0 \\ 0 + 0 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

6. $x^2 + 10x = -25$



So, the solution is $x = -5$.

Check:

$x = -5$:

$$\begin{aligned} x^2 + 10x &= -25 \\ (-5)^2 + 10(-5) &\stackrel{?}{=} -25 \\ 25 + 10(-5) &\stackrel{?}{=} -25 \\ 25 - 50 &\stackrel{?}{=} -25 \\ -25 &= -25 \checkmark \end{aligned}$$

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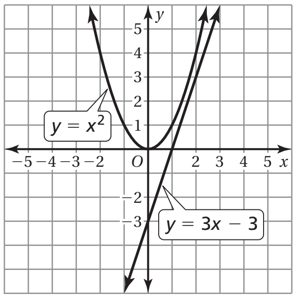
7. $x^2 = 3x - 3$

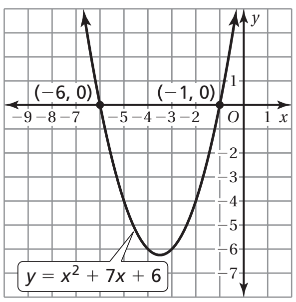
$y = x^2$

$y = 3x - 3$



The graphs do not intersect. So, $x^2 = 3x - 3$ has no real solutions.

8. $x^2 + 7x = -6$



So, the solutions are $x = -6$ and $x = -1$.

Check:

$x = -6$:

$$\begin{aligned} x^2 + 7x &= -6 \\ (-6)^2 + 7(-6) &\overset{?}{=} -6 \\ 36 + 7(-6) &\overset{?}{=} -6 \\ 36 - 42 &\overset{?}{=} -6 \\ -6 &= -6 \checkmark \end{aligned}$$

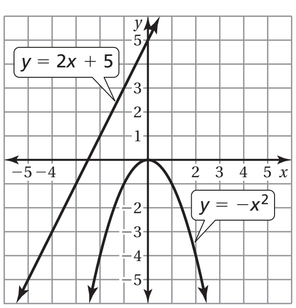
$x = -1$:

$$\begin{aligned} x^2 + 7x &= -6 \\ (-1)^2 + 7(-1) &\overset{?}{=} -6 \\ 1 + 7(-1) &\overset{?}{=} -6 \\ 1 - 7 &\overset{?}{=} -6 \\ -6 &= -6 \checkmark \end{aligned}$$

9. $2x + 5 = -x^2$

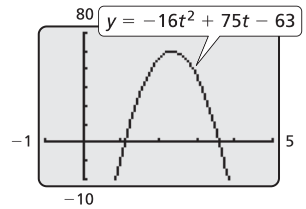
$y = 2x + 5$

$y = -x^2$



The graphs do not intersect. So, $2x + 5 = -x^2$ has no real solutions.

10.
$$\begin{aligned} -16t^2 + 75t + 2 &= 65 \\ -65 \quad &\quad -65 \\ -16t^2 + 75t - 63 &= 0 \end{aligned}$$



Use the *zero* feature to find that the football is 65 feet above the ground after about 1.1 seconds and about 3.6 seconds.

9.1 Exercises (pp. 459–461)

Vocabulary and Concept Check

1. A quadratic equation is a nonlinear equation that can be written in the standard form

$ax^2 + bx + c = 0$, where $a \neq 0$.

2. The equation $x^2 + x - 4 = 0$ does not belong. It is the only equation in standard form.

3. You can use a graph to find the number of solutions of a quadratic equation by finding the x-intercepts.

4. The roots, or solutions, of an equation are the same as the zeros of the related function or the x-intercepts of its graph.

Practice and Problem Solving

5. $x^2 - 10x + 24 = 0$

The solutions are $x = 4$ and $x = 6$.

Check:

$x = 4$:

$$\begin{aligned} x^2 - 10x + 24 &= 0 \\ 4^2 - 10(4) + 24 &\overset{?}{=} 0 \\ 16 - 10(4) + 24 &\overset{?}{=} 0 \\ 16 - 40 + 24 &\overset{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

$x = 6$:

$$\begin{aligned} x^2 - 10x + 24 &= 0 \\ 6^2 - 10(6) + 24 &\overset{?}{=} 0 \\ 36 - 10(6) + 24 &\overset{?}{=} 0 \\ 36 - 60 + 24 &\overset{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

6. $-x^2 - 4x - 6 = 0$

The graph does not intersect the x-axis. So, $-x^2 - 4x - 6 = 0$ has no real solutions.

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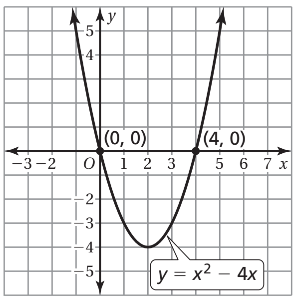
7. $x^2 + 12x + 36 = 0$

The solution is $x = -6$.

Check:

$$x = -6: \quad \begin{aligned} x^2 + 12x + 36 &= 0 \\ (-6)^2 + 12(-6) + 36 &\stackrel{?}{=} 0 \\ 36 + 12(-6) + 36 &\stackrel{?}{=} 0 \\ 36 - 72 + 36 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

8. $x^2 - 4x = 0$



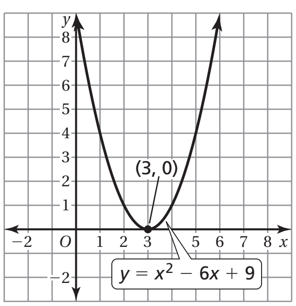
So, the solutions are $x = 0$ and $x = 4$.

Check:

$$x = 0: \quad \begin{aligned} x^2 - 4x &= 0 \\ 0^2 - 4(0) &\stackrel{?}{=} 0 \\ 0 - 4(0) &\stackrel{?}{=} 0 \\ 0 - 0 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

$$x = 4: \quad \begin{aligned} x^2 - 4x &= 0 \\ 4^2 - 4(4) &\stackrel{?}{=} 0 \\ 16 - 4(4) &\stackrel{?}{=} 0 \\ 16 - 16 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

9. $x^2 - 6x + 9 = 0$

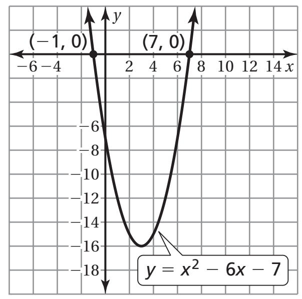


So, the solution is $x = 3$.

Check:

$$x = 3: \quad \begin{aligned} x^2 - 6x + 9 &= 0 \\ 3^2 - 6(3) + 9 &\stackrel{?}{=} 0 \\ 9 - 6(3) + 9 &\stackrel{?}{=} 0 \\ 9 - 18 + 9 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

10. $x^2 - 6x - 7 = 0$



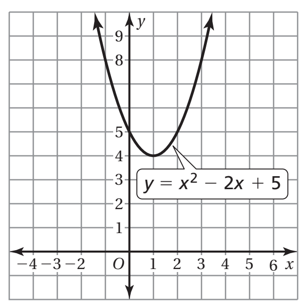
So, the solutions are $x = -1$ and $x = 7$.

Check:

$$x = -1: \quad \begin{aligned} x^2 - 6x - 7 &= 0 \\ (-1)^2 - 6(-1) - 7 &\stackrel{?}{=} 0 \\ 1 - 6(-1) - 7 &\stackrel{?}{=} 0 \\ 1 + 6 - 7 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

$$x = 7: \quad \begin{aligned} x^2 - 6x - 7 &= 0 \\ 7^2 - 6(7) - 7 &\stackrel{?}{=} 0 \\ 49 - 6(7) - 7 &\stackrel{?}{=} 0 \\ 49 - 42 - 7 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

11. $x^2 - 2x + 5 = 0$



The graph does not intersect the x-axis. So, $x^2 - 2x + 5 = 0$ has no real solutions.

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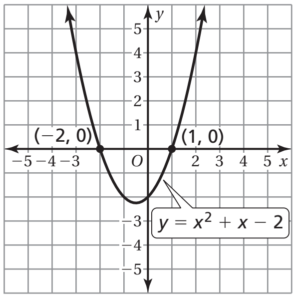
12. $x^2 + x - 2 = 0$



So, the solutions are $x = -2$ and $x = 1$.

Check:

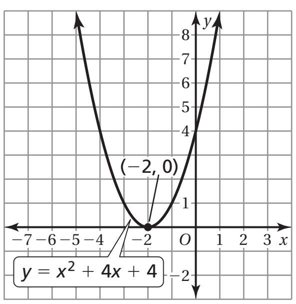
$x = -2$:

$$\begin{aligned} x^2 + x - 2 &= 0 \\ (-2)^2 + (-2) - 2 &\stackrel{?}{=} 0 \\ 4 + (-2) - 2 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

$x = 1$:

$$\begin{aligned} x^2 + x - 2 &= 0 \\ 1^2 + 1 - 2 &\stackrel{?}{=} 0 \\ 1 + 1 - 2 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

13. $x^2 + 4x + 4 = 0$



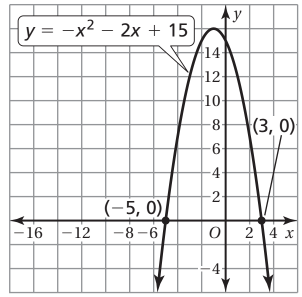
So, the solution is $x = -2$.

Check:

$x = -2$:

$$\begin{aligned} x^2 + 4x + 4 &= 0 \\ (-2)^2 + 4(-2) + 4 &\stackrel{?}{=} 0 \\ 4 + 4(-2) + 4 &\stackrel{?}{=} 0 \\ 4 - 8 + 4 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

14. $-x^2 - 2x + 15 = 0$



So, the solutions are $x = -5$ and $x = 3$.

Check:

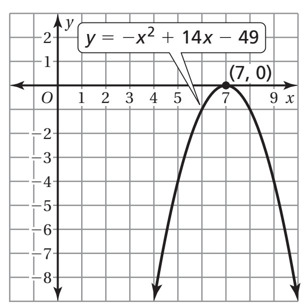
$x = -5$:

$$\begin{aligned} -x^2 - 2x + 15 &= 0 \\ -(-5)^2 - 2(-5) + 15 &\stackrel{?}{=} 0 \\ -25 - 2(-5) + 15 &\stackrel{?}{=} 0 \\ -25 + 10 + 15 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

$x = 3$:

$$\begin{aligned} -x^2 - 2x + 15 &= 0 \\ -(3^2) - 2(3) + 15 &\stackrel{?}{=} 0 \\ -9 - 2(3) + 15 &\stackrel{?}{=} 0 \\ -9 - 6 + 15 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

15. $-x^2 + 14x - 49 = 0$



So, the solution is $x = 7$.

Check:

$x = 7$:

$$\begin{aligned} -x^2 + 14x - 49 &= 0 \\ -(7)^2 + 14(7) - 49 &\stackrel{?}{=} 0 \\ -49 + 14(7) - 49 &\stackrel{?}{=} 0 \\ -49 + 98 - 49 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

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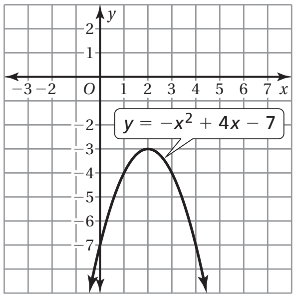
16. $-x^2 + 4x - 7 = 0$



The graph does not intersect the x-axis. So, $-x^2 + 4x - 7 = 0$ has no real solutions.

17. $y = -x^2 + 5x$

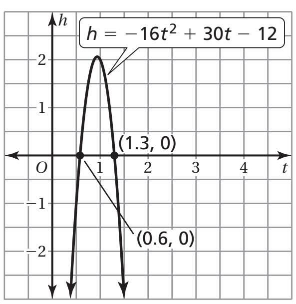
a. The x-intercepts represent the horizontal position of the ball where it is struck and where it lands.

b. The ball lands 5 yards away because the second x-intercept is $x = 5$.

18. $h = -16t^2 + 30t + 4$

$h = 16$

$$\begin{aligned} 16 &= -16t^2 + 30t + 4 \\ \underline{-16} & \qquad\qquad\qquad\quad \underline{-16} \\ 0 &= -16t^2 + 30t - 12 \end{aligned}$$

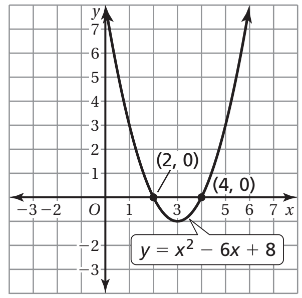


So, the solutions are $t = 0.6$ and $t = 1.3$.

The ball is 16 feet above the ground after about 0.6 second and 1.3 seconds.

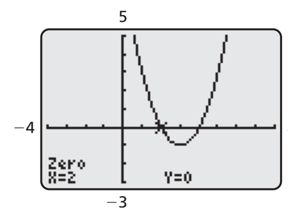
19.

$$\begin{aligned} x^2 &= 6x - 8 \\ \underline{-6x} & \quad \underline{-6x} \\ x^2 - 6x &= -8 \\ \underline{+8} & \quad \underline{+8} \\ x^2 - 6x + 8 &= 0 \end{aligned}$$



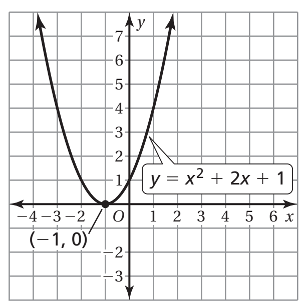
So, the solutions are $x = 2$ and $x = 4$.

Check:



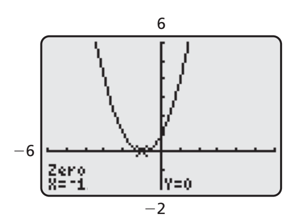
20.

$$\begin{aligned} x^2 &= -1 - 2x \\ \underline{+2x} & \qquad\quad \underline{+2x} \\ x^2 + 2x &= -1 \\ \underline{+1} & \quad \underline{+1} \\ x^2 + 2x + 1 &= 0 \end{aligned}$$



So, the solution is $x = -1$.

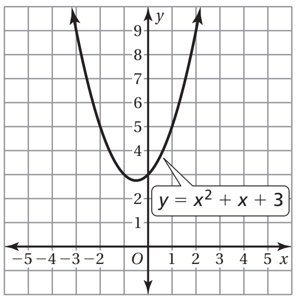
Check:



Chapter 9

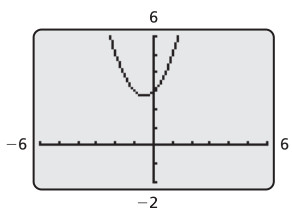
21.

$$
\begin{aligned}
x^2 &= -x - 3\\
+x\quad &\quad +x\\
x^2 + x &= -3\\
+3\quad &\quad +3\\
x^2 + x + 3 &= 0
\end{aligned}
$$



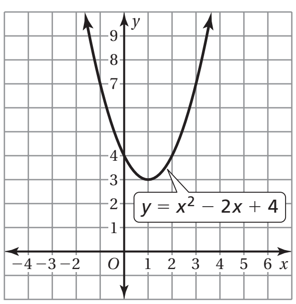
The graph does not intersect the x-axis. So, $x^2 = -x - 3$ has no real solutions.

Check:



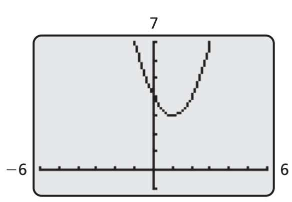
22.

$$
\begin{aligned}
x^2 &= 2x - 4\\
-2x\quad &\quad -2x\\
x^2 - 2x &= -4\\
+4\quad &\quad +4\\
x^2 - 2x + 4 &= 0
\end{aligned}
$$



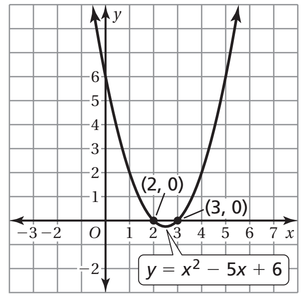
The graph does not intersect the x-axis. So, $x^2 = 2x - 4$ has no real solutions.

Check:



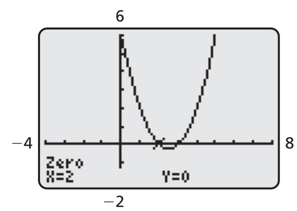
23.

$$
\begin{aligned}
5x - 6 &= x^2\\
-x^2\quad &\quad -x^2\\
-x^2 + 5x - 6 &= 0\\
x^2 - 5x + 6 &= 0
\end{aligned}
$$



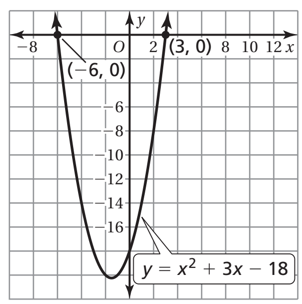
So, the solutions are $x = 2$ and $x = 3$.

Check:



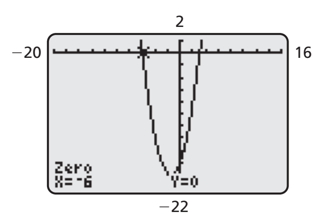
24.

$$
\begin{aligned}
3x - 18 &= -x^2\\
+x^2\quad &\quad +x^2\\
x^2 + 3x - 18 &= 0
\end{aligned}
$$



So, the solutions are $x = -6$ and $x = 3$.

Check:

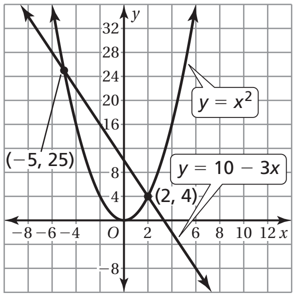


Chapter 9

25. $x^2 = 10 - 3x$

$y = x^2$

$y = 10 - 3x$



So, the solutions are $x = -5$ and $x = 2$.

Check:

$x = -5$:

$$x^2 = 10 - 3x$$
$$(-5)^2 \stackrel{?}{=} 10 - 3(-5)$$
$$25 \stackrel{?}{=} 10 + 15$$
$$25 = 25 \checkmark$$

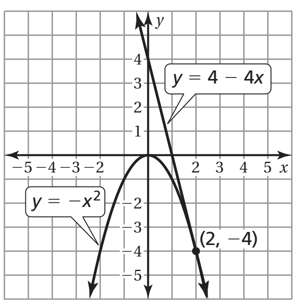
$x = 2$:

$$x^2 = 10 - 3x$$
$$2^2 \stackrel{?}{=} 10 - 3(2)$$
$$4 \stackrel{?}{=} 10 - 6$$
$$4 = 4 \checkmark$$

26. $4 - 4x = -x^2$

$y = 4 - 4x$

$y = -x^2$



So, the solution is $x = 2$.

Check:

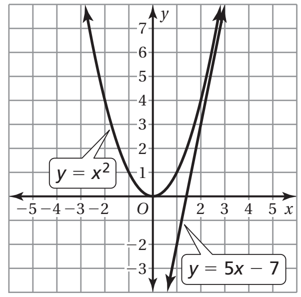
$x = 2$: $4 - 4x = -x^2$

$$4 - 4(2) \stackrel{?}{=} -(2)^2$$
$$4 - 8 \stackrel{?}{=} -4$$
$$-4 = -4 \checkmark$$

27. $5x - 7 = x^2$

$y = 5x - 7$

$y = x^2$

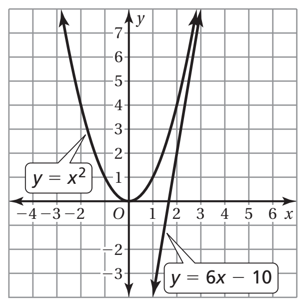


The graphs do not intersect. So, $5x - 7 = x^2$ has no real solutions.

28. $x^2 = 6x - 10$

$y = x^2$

$y = 6x - 10$

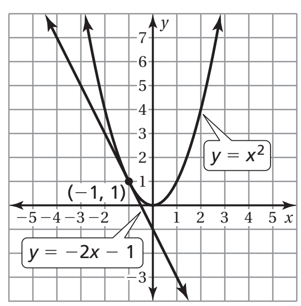


The graphs do not intersect. So, $x^2 = 6x - 10$ has no real solutions.

29. $x^2 = -2x - 1$

$y = x^2$

$y = -2x - 1$



So, the solution is $x = -1$.

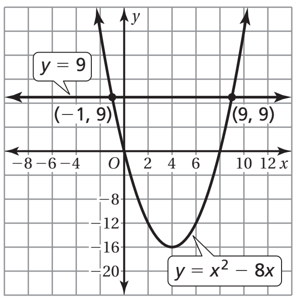
Check:

$x = -1$:
$$x^2 = -2x - 1$$
$$(-1)^2 \stackrel{?}{=} -2(-1) - 1$$
$$1 \stackrel{?}{=} 2 - 1$$
$$1 = 1 \checkmark$$

30. $x^2 - 8x = 9$

$y = x^2 - 8x$

$y = 9$



So, the solutions are $x = -1$ and $x = 9$.

Check:

$x = -1$:
$$x^2 - 8x = 9$$
$$(-1)^2 - 8(-1) \stackrel{?}{=} 9$$
$$1 + 8 \stackrel{?}{=} 9$$
$$9 = 9 \checkmark$$

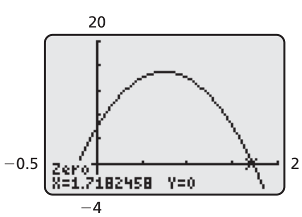
$x = 9$:
$$x^2 - 8x = 9$$
$$9^2 - 8(9) \stackrel{?}{=} 9$$
$$81 - 72 \stackrel{?}{=} 9$$
$$9 = 9 \checkmark$$

31. *Sample answer:* Method 1 is preferable because it involves graphing just one equation. If the graph crosses the x-axis, it has a solution. The solutions to the equation are the x-intercepts.

32. The solver identified the value of the y-intercept as the solution rather than the x-intercept. The solution is $x = -3$.

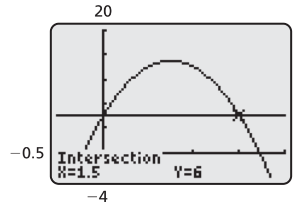
33. $h = -16t^2 + 24t + 6$

a.



The ball is in the air about 1.7 seconds.

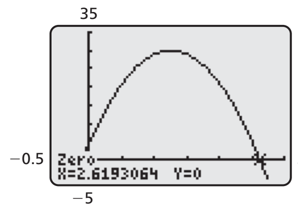
b.



The ball is above 6 feet for 1.5 seconds.

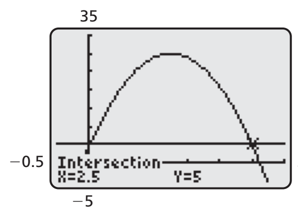
34. $h = -16t^2 + 40t + 5$

a.



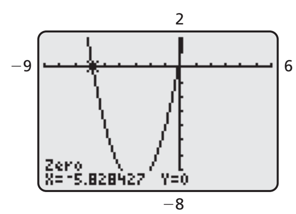
The ball is in the air about 2.6 seconds.

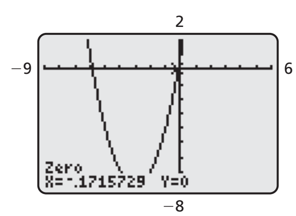
b.



If you catch the ball at a height of 5 feet, then it is in the air for 2.5 seconds.

35. $f(x) = x^2 + 6x + 1$

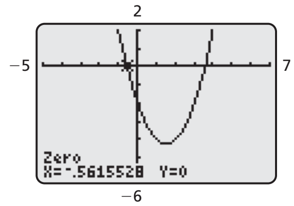


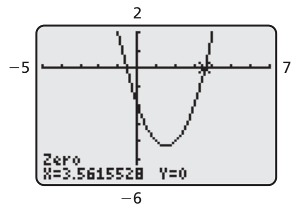


So, the zeros are $x \approx -5.8$ and $x \approx -0.2$.

Chapter 9

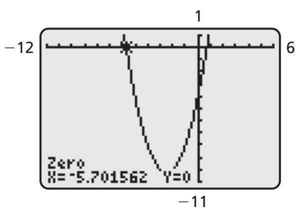
36. $f(x) = x^2 - 3x - 2$

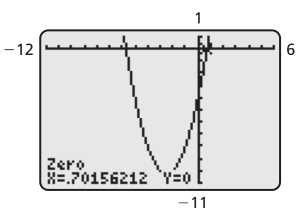




So, the zeros are $x \approx -0.6$ and $x \approx 3.6$.

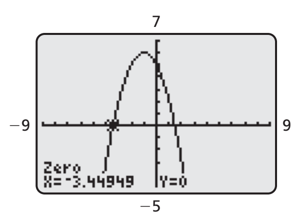
37. $f(x) = x^2 + 5x - 4$

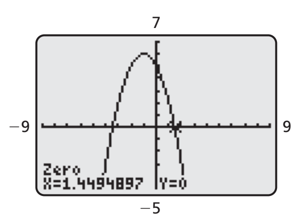




So, the zeros are $x \approx -5.7$ and $x \approx 0.7$.

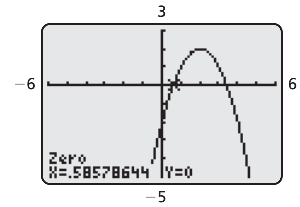
38. $f(x) = -x^2 - 2x + 5$

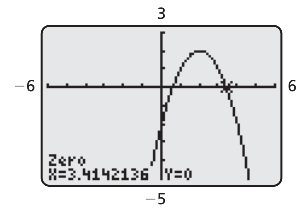




So, the zeros are $x \approx -3.4$ and $x \approx 1.4$.

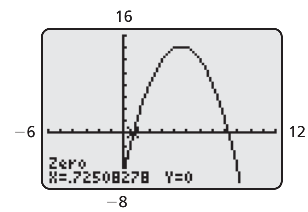
39. $f(x) = -x^2 + 4x - 2$

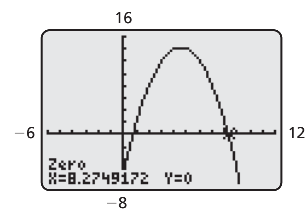




So, the zeros are $x \approx 0.6$ and $x \approx 3.4$.

40. $f(x) = -x^2 + 9x - 6$



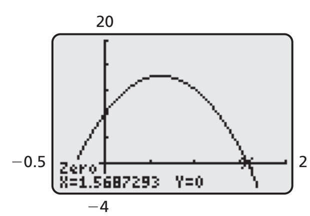


So, the zeros are $x \approx 0.7$ and $x \approx 8.3$.

41. **a.** $h = at^2 + bt + c, a = -16, b = 20, c = 8$

$h = -16t^2 + 20t + 8$

b.

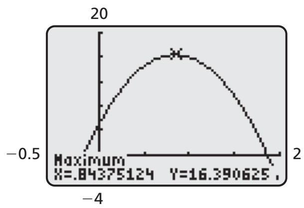


So, the dirt bike lands after about 1.6 seconds.

Chapter 9

42. a. $h = at^2 + bt + c$; $a = -16, b = 27, c = 5$

$h = -16t^2 + 27t + 5$



The maximum of the graph is at $(0.8, 16.4)$. The wall is 16.5 feet high. So, the throw is not high enough to clear the wall.

b. Yes, the height of the competitors does factor into their success at this event. By throwing the keg at the same velocity from different heights, the maximum height of the keg is affected.

43. This statement is sometimes true. The value of c determines the y-intercept of the graph. When c is negative, the graph has no x-intercepts. When c is zero, the graph has one x-intercept. When c is positive, the graph has two x-intercepts.

44. This statement is always true. The sign of a determines whether the parabola opens up (if a is positive) or down (if a is negative). The sign of c determines whether the y-intercept is positive or negative. So, if the signs are the same, then the parabola will not have x-intercepts.

45. This statement is never true. A quadratic equation has at most two zeros.

Fair Game Review

46. $4\sqrt{36} = 4\sqrt{6^2} = 4(6) = 24$

47. $9\sqrt{81} = 9\sqrt{9^2} = 9(9) = 81$

48. $2\sqrt{27} = 2\sqrt{9 \bullet 3} = 2\sqrt{9} \bullet \sqrt{3} = 2(3)\sqrt{3} = 6\sqrt{3}$

49. $5\sqrt{50} = 5\sqrt{25 \bullet 2}$

$= 5\sqrt{25} \bullet \sqrt{2}$

$= 5(5)\sqrt{2}$

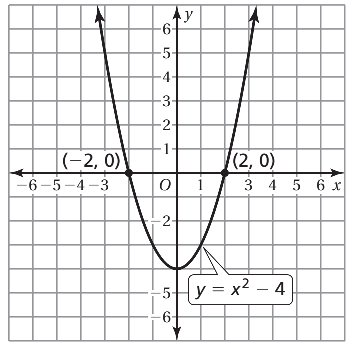
$= 25\sqrt{2}$

50. D; $\left(\dfrac{2x^3}{3m^5}\right)^2 = \dfrac{2^2(x^3)^2}{3^3(m^5)^2} = \dfrac{4x^6}{9m^{10}}$

Section 9.2

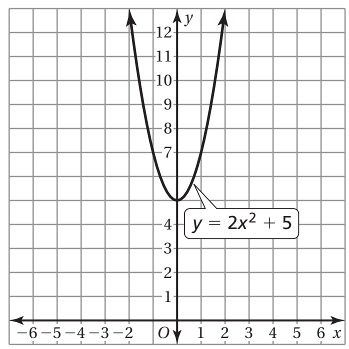
9.2 Activity (pp. 462–463)

1. a. $x^2 - 4 = 0$



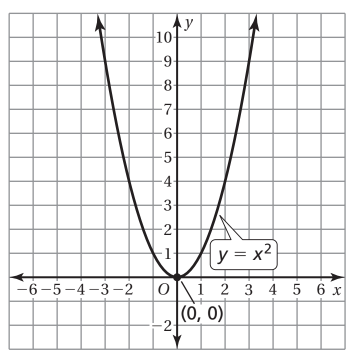
So, the solutions are $x = -2$ and $x = 2$.

b. $2x^2 + 5 = 0$



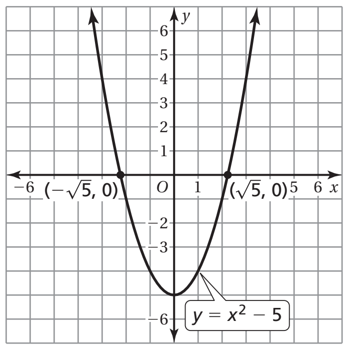
The graph does not intersect the x-axis. So, $2x^2 + 5 = 0$ has no real solutions.

c. $x^2 = 0$



So, the solution is $x = 0$.

d. $x^2 - 5 = 0$



So, the solutions are $x = -\sqrt{5}$ and $x = \sqrt{5}$.

The number of solutions of $ax^2 + c = 0$ equals the number of x-intercepts of $y = ax^2 + c$.

2. a.

x	$x^2 - 5$
2.21	–0.1159
2.22	–0.0716
2.23	–0.0271
2.24	0.0176
2.25	0.0625
2.26	0.1076

b.

x	$x^2 - 5$
–2.21	–0.1159
–2.22	–0.0716
–2.23	–0.0271
–2.24	0.0176
–2.25	0.0625
–2.26	0.1076

So, the solutions are about ± 2.235 because the tables show that one solution is between 2.23 and 2.24 and one solution is between –2.23 and –2.24.

3. a. Yes;

$$\begin{aligned} x^2 - 5 &= 0 \\ +5 \quad & \quad +5 \\ x^2 &= 5 \end{aligned}$$

So, $x^2 - 5 = 0$ and $x^2 = 5$ are equivalent.

b.

$$\begin{aligned} x^2 - 5 &= 0 \\ +5 \quad & \quad +5 \\ x^2 &= 5 \\ \sqrt{x^2} &= \sqrt{5} \\ x &\approx \pm 2.236 \end{aligned}$$

So, the estimates in Activity 2 are off by about 0.001.

c.

$$\begin{aligned} x^2 - 5 &= 0 \\ +5 \quad & \quad +5 \\ x^2 &= 5 \\ \sqrt{x^2} &= \sqrt{5} \\ x &= \pm\sqrt{5} \end{aligned}$$

4. You can determine the number of solutions of a quadratic equation of the form $ax^2 + c = 0$ by graphing the equation, making a table of values, or solving the equation for x.

5. a.

$$\begin{aligned} x^2 - 2 &= 0 \\ +2 \quad & \quad +2 \\ x^2 &= 2 \\ x &= \pm\sqrt{2} \end{aligned}$$

The exact solutions are $x = \sqrt{2}$ and $x = -\sqrt{2}$.

The estimated solutions are $x \approx 1.414$ and $x \approx -1.414$.

b.

$$\begin{aligned} 3x^2 - 15 &= 0 \\ +15 \quad & \quad +15 \\ 3x^2 &= 15 \\ \frac{3x^2}{3} &= \frac{15}{3} \\ x^2 &= 5 \\ x &= \pm\sqrt{5} \end{aligned}$$

The exact solutions are $x = \sqrt{5}$ and $x \approx -\sqrt{5}$.

The estimated solutions are $x \approx 2.236$ and $x \approx -2.236$.

c.

$$\begin{aligned} x^2 &= 8 \\ x &= \pm\sqrt{8} \\ x &= \pm\sqrt{4 \bullet 2} \\ x &= \pm 2\sqrt{2} \end{aligned}$$

The exact solutions are $x = 2\sqrt{2}$ and $x = -2\sqrt{2}$.

The estimated solutions are $x \approx 2.828$ and $x \approx -2.828$.

Chapter 9

9.2 On Your Own (pp. 464–465)

1. $$\begin{aligned} -3x^2 &= -75 \\ \frac{-3x^2}{-3} &= \frac{-75}{-3} \\ x^2 &= 25 \\ x &= \pm\sqrt{25} \\ x &= \pm 5 \end{aligned}$$

The solutions are $x = 5$ and $x = -5$.

2. $$\begin{aligned} x^2 + 12 &= 10 \\ -12 \quad & \quad -12 \\ x^2 &= -2 \end{aligned}$$

The equation has no real solutions.

3. $$\begin{aligned} 4x^2 - 15 &= -15 \\ +15 \quad & \quad +15 \\ 4x^2 &= 0 \\ \frac{4x^2}{4} &= \frac{0}{4} \\ x^2 &= 0 \\ x &= \sqrt{0} \\ x &= 0 \end{aligned}$$

The only solution is $x = 0$.

4. $$\begin{aligned} (x + 7)^2 &= 0 \\ x + 7 &= 0 \\ -7 \quad & \quad -7 \\ x &= -7 \end{aligned}$$

The only solution is $x = -7$.

5. $$\begin{aligned} 4(x - 3)^2 &= 9 \\ \frac{4(x - 3)^2}{4} &= \frac{9}{4} \\ (x - 3)^2 &= \frac{9}{4} \\ x - 3 &= \pm\sqrt{\frac{9}{4}} \\ x - 3 &= \pm\frac{3}{2} \\ x &= 3 \pm \frac{3}{2} \end{aligned}$$

So, the solutions are $x = 3 + \frac{3}{2} = 4.5$ and $x = 3 - \frac{3}{2} = 1.5$.

6. $$\begin{aligned} (2x + 1)^2 &= 35 \\ 2x + 1 &= \pm\sqrt{35} \\ -1 \quad & \quad -1 \\ 2x &= \pm\sqrt{35} - 1 \\ \frac{2x}{2} &= \frac{\pm\sqrt{35} - 1}{2} \\ x &= \frac{-1 \pm \sqrt{35}}{2} \end{aligned}$$

So, the solutions are $x = \frac{-1 + \sqrt{35}}{2} \approx 2.458$ and $x = \frac{-1 - \sqrt{35}}{2} \approx -3.458$.

7. $V = \ell wh;\ \ell = 3w,\ h = 3,\ V = 315$

$$\begin{aligned} 315 &= (3w)(w)(3) \\ 315 &= 9w^2 \\ \frac{315}{9} &= \frac{9w^2}{9} \\ 35 &= w^2 \\ \pm\sqrt{35} &= w \end{aligned}$$

The solutions are $\sqrt{35}$ and $-\sqrt{35}$. Use the positive solution.

So, the width is $\sqrt{35} \approx 5.9$ feet and the length is $3\sqrt{35} \approx 17.7$ feet.

9.2 Exercises (pp. 466–467)

Vocabulary and Concept Check

1. When d is positive, the equation $x^2 = d$ has two real solutions. When d is 0 the equation $x^2 = d$ has one real solution, 0. When d is negative, the equation $x^2 = d$ has no real solutions.

2. The equation $x^2 = -7$ does not belong because it has no real solutions while each of the other three equations has two real solutions.

Practice and Problem Solving

3. $x^2 - 11 = 0$

The equation has two real solutions.

$$\begin{aligned} x^2 - 11 &= 0 \\ +11 \quad & \quad +11 \\ x^2 &= 11 \\ x &= \pm\sqrt{11} \end{aligned}$$

So, $x \approx 3.317$ and $x \approx -3.317$.

4. $x^2 + 10 = 0$

The equation has no real solutions.

Chapter 9

5. $2x^2 - 3 = 0$

The equation has two real solutions.

$$\begin{aligned} 2x^2 - 3 &= 0 \\ +3 \quad &\quad +3 \\ 2x^2 &= 3 \\ \frac{2x^2}{2} &= \frac{3}{2} \\ x^2 &= \frac{3}{2} \\ x &= \pm\sqrt{\frac{3}{2}} \end{aligned}$$

So, $x \approx 1.225$ and $x \approx -1.225$.

6. $x^2 = 25$

$d = 25$

Because d is positive, the equation has two solutions.

$$\begin{aligned} x^2 &= 25 \\ x &= \pm 5 \end{aligned}$$

The solutions are $x = 5$ and $x = -5$.

7. $x^2 = -36$

$d = -36$

Because d is negative, the equation has no real solutions.

8. $x^2 = 8$

$d = 8$

Because d is positive, the equation has two solutions.

$$\begin{aligned} x^2 &= 8 \\ x &= \pm\sqrt{8} \\ x &= \pm\sqrt{4 \bullet 2} \\ x &= \pm 2\sqrt{2} \end{aligned}$$

The solutions are $x = 2\sqrt{2} \approx 2.828$ and $x = -2\sqrt{2} \approx -2.828$.

9. $x^2 = 21$

$d = 21$

Because d is positive, the equation has two solutions.

$$\begin{aligned} x^2 &= 21 \\ x &= \pm\sqrt{21} \end{aligned}$$

The solutions are $x = \sqrt{21} \approx 4.583$ and $x = -\sqrt{21} \approx -4.583$.

10. $x^2 = 0$

$d = 0$

Because d is zero, the equation has one solution.

$$\begin{aligned} x^2 &= 0 \\ x &= 0 \end{aligned}$$

The only solution is $x = 0$.

11. $x^2 = 169$

$d = 169$

Because d is positive, the equation has two solutions.

$$\begin{aligned} x^2 &= 169 \\ x &= \pm 13 \end{aligned}$$

The solutions are $x = 13$ and $x = -13$.

12. $x^2 - 16 = 0$

$$\begin{aligned} x^2 - 16 &= 0 \\ +16 \quad &\quad +16 \\ x^2 &= 16 \\ x &= \pm\sqrt{16} \\ x &= \pm 4 \end{aligned}$$

The solutions are $x = 4$ and $x = -4$.

13. $x^2 + 12 = 0$

$$\begin{aligned} x^2 + 12 &= 0 \\ -12 \quad &\quad -12 \\ x^2 &= -12 \end{aligned}$$

The equation has no real solutions.

14. $x^2 + 6 = 0$

$$\begin{aligned} x^2 + 6 &= 0 \\ -6 \quad &\quad -6 \\ x^2 &= -6 \end{aligned}$$

The equation has no real solutions.

15. $x^2 - 61 = 0$

$$\begin{aligned} x^2 - 61 &= 0 \\ +61 \quad &\quad +61 \\ x^2 &= 61 \\ x &= \pm\sqrt{61} \end{aligned}$$

The solutions are $x = \sqrt{61} \approx 7.810$ and $x = -\sqrt{61} \approx -7.810$.

16. $2x^2 - 98 = 0$

$$\begin{aligned} 2x^2 - 98 &= 0 \\ +98 \quad &\quad +98 \\ 2x^2 &= 98 \\ \frac{2x^2}{2} &\quad \frac{98}{2} \\ x^2 &= 49 \\ x &= \pm\sqrt{49} \\ x &= \pm 7 \end{aligned}$$

The solutions are $x = 7$ and $x = -7$.

Chapter 9

17.
$$\begin{aligned} -x^2 + 9 &= 9 \\ \underline{-9} &\quad \underline{-9} \\ -x^2 &= 0 \\ \frac{-x^2}{-1} &= \frac{0}{-1} \\ x^2 &= 0 \\ x &= 0 \end{aligned}$$

The only solution is $x = 0$.

18.
$$\begin{aligned} x^2 + 13 &= 7 \\ \underline{-13} &\quad \underline{-13} \\ x^2 &= -6 \end{aligned}$$

The equation has no real solutions.

19.
$$\begin{aligned} -4x^2 - 5 &= -5 \\ \underline{+5} &\quad \underline{+5} \\ 4x^2 &= 0 \\ \frac{4x^2}{4} &= \frac{0}{4} \\ x^2 &= 0 \\ x &= 0 \end{aligned}$$

The only solution is $x = 0$.

20.
$$\begin{aligned} -3x^2 + 8 &= 8 \\ \underline{-8} &\quad \underline{-8} \\ -3x^2 &= 0 \\ \frac{-3x^2}{-3} &= \frac{0}{-3} \\ x^2 &= 0 \\ x &= 0 \end{aligned}$$

The only solution is $x = 0$.

21. The solver omitted the second solution of the equation, $x = -6$.

$$\begin{aligned} 2x^2 - 33 &= 39 \\ \underline{+33} &\quad \underline{+33} \\ 2x^2 &= 72 \\ \frac{2x^2}{2} &= \frac{72}{2} \\ x^2 &= 36 \\ x &= \pm\sqrt{36} \\ x &= \pm 6 \end{aligned}$$

The solutions are $x = 6$ and $x = -6$.

22. $h = 0$

$$\begin{aligned} h &= -16x^2 + 16 \\ 0 &= -16x^2 + 16 \\ \underline{-16} &\qquad\quad \underline{-16} \\ -16 &= -16x^2 \\ \frac{-16}{-16} &= \frac{-16x^2}{-16} \\ 1 &= x^2 \\ \pm\sqrt{1} &= x \\ \pm 1 &= x \end{aligned}$$

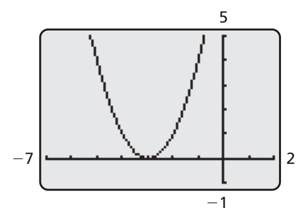
Use the positive solution.

The box hits the floor after 1 second.

23.
$$\begin{aligned} (x + 3)^2 &= 0 \\ x + 3 &= 0 \\ \underline{-3} &\quad \underline{-3} \\ x &= -3 \end{aligned}$$

The only solution is $x = -3$.

Check: Graph the function $y = (x + 3)^2$.

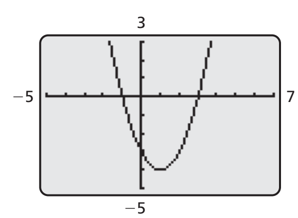


The only zero is $x = -3$, so the solution checks.

24.
$$\begin{aligned} (x - 1)^2 &= 4 \\ x - 1 &= \pm 2 \\ \underline{+1} &\quad \underline{+1} \\ x &= \pm 2 + 1 \end{aligned}$$

So, the solutions are $x = 2 + 1 = 3$ and $x = -2 + 1 = -1$.

Check: Graph the function $y = (x - 1)^2 - 4$.



The zeros are -1 and 3, so the solution checks.

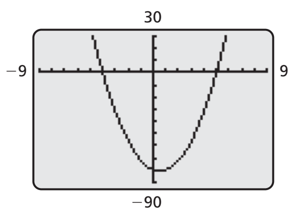
Chapter 9

25.
$$\begin{aligned}(2x-1)^2 &= 81\\ 2x-1 &= \pm 9\\ \underline{+1} &\quad \underline{+1}\\ 2x &= \pm 9 + 1\\ \frac{2x}{2} &= \frac{\pm 9 + 1}{2}\\ x &= \pm\frac{9}{2} + \frac{1}{2}\end{aligned}$$

So, the solutions are $x = \frac{9}{2} + \frac{1}{2} = 5$ and

$x = -\frac{9}{2} + \frac{1}{2} = -4.$

Check: Graph the function $y = (2x-1)^2 - 81.$



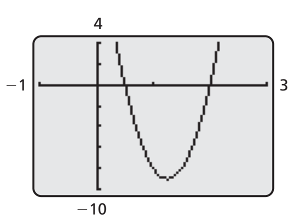
The zeros are -4 and 5, so the solution checks.

26.
$$\begin{aligned}(4x-5)^2 &= 9\\ 4x-5 &= \pm 3\\ \underline{+5} &\quad \underline{+5}\\ 4x &= \pm 3 + 5\\ \frac{4x}{4} &= \frac{\pm 3 + 5}{4}\\ x &= \frac{\pm 3}{4} + \frac{5}{4}\end{aligned}$$

So, the solutions are $x = \frac{3}{4} + \frac{5}{4} = 2$ and

$x = -\frac{3}{4} + \frac{5}{4} = \frac{1}{2}.$

Check: Graph the function $y = (4x-5)^2 - 9.$



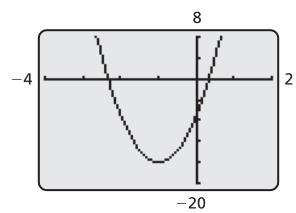
The zeros are $\frac{1}{2}$ and 2, so the solution checks.

27.
$$\begin{aligned}9(x+1)^2 &= 16\\ (x+1)^2 &= \frac{16}{9}\\ x+1 &= \pm\frac{4}{3}\\ \underline{-1} &\quad \underline{-1}\\ x &= \pm\frac{4}{3} - 1\end{aligned}$$

So, the solutions are $x = \frac{4}{3} - 1 = \frac{1}{3}$ and

$x = -\frac{4}{3} - 1 = -\frac{7}{3}.$

Check: Graph the function $y = 9(x+1)^2 - 16.$



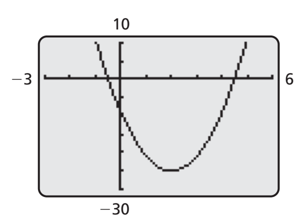
The zeros are about -2.333 and about 0.333, so the solution checks.

28.
$$\begin{aligned}4(x-2)^2 &= 25\\ (x-2)^2 &= \frac{25}{4}\\ x-2 &= \pm\frac{5}{2}\\ \underline{+2} &\quad \underline{+2}\\ x &= \pm\frac{5}{2} + 2\end{aligned}$$

So, the solutions are $x = \frac{5}{2} + 2 = \frac{9}{2}$ and

$x = -\frac{5}{2} + 2 = -\frac{1}{2}.$

Check: Graph the function $y = 4(x-2)^2 - 25.$



The zeros are -0.5 and 4.5, so the solution checks.

29. $A = 64 \text{ in.}^2$

$s = x$

$$\begin{aligned} A &= s^2 \\ 64 &= x^2 \\ \pm\sqrt{64} &= x \\ \pm 8 &= x \end{aligned}$$

The solutions are $x = 8$ and $x = -8$. Use the positive solution.

So, the side length of the square is 8 inches.

30. $A = 78 \text{ cm}^2$

$\ell = 3x, w = 2x$

$$\begin{aligned} A &= \ell w \\ 78 &= (3x)(2x) \\ 78 &= 6x^2 \\ 13 &= x^2 \\ \pm\sqrt{13} &= x \end{aligned}$$

The solutions are $\sqrt{13}$ and $-\sqrt{13}$. Use the positive solution.

So, the width is $2\sqrt{13} \approx 7.2$ centimeters and the length is $3\sqrt{13} \approx 10.8$ centimeters.

31. $A = 144\pi \text{ ft}^2$

$r = x$

$$\begin{aligned} A &= \pi r^2 \\ 144\pi &= \pi x^2 \\ 144 &= x^2 \\ \pm\sqrt{144} &= x \\ \pm 12 &= x \end{aligned}$$

The solutions are $x = 12$ and $x = -12$. Use the positive solution.

So, the radius of the circle is 12 feet.

32. $V = 33{,}000 \text{ in.}^3, \ell = 2x, w = x, h = 24 \text{ in.}$

$$\begin{aligned} V &= \ell wh \\ 33{,}000 &= (2x)(x)(24) \\ 33{,}000 &= 48x^2 \\ 687.5 &= x^2 \\ \pm\sqrt{687.5} &= x \end{aligned}$$

The solutions are $x = \sqrt{687.5}$ and $x = -\sqrt{687.5}$. Use the positive solution.

So, the length is $2\sqrt{687.5} \approx 52.4$ inches and the width is $\sqrt{687.5} \approx 26.2$ inches.

33. Area of the rug $= 6^2 = 36 \text{ ft}^2$

$A =$ area of inner square

$x =$ length of inner square side length

$$\begin{aligned} A = (.25)(36) &= x^2 \\ 9 &= x^2 \\ \pm 3 &= x \end{aligned}$$

The solutions are $x = 3$ and $x = -3$. Use the positive solution.

So, the side length is 3 feet.

34. You can approximate the roots of a quadratic equation when the roots are not integers by finding the zeros of the graph of the function.

35. $ax^2 + c = 0$

a. The equation will have two solutions when a is positive and c is negative or when a is negative and c is positive.

b. The equation will have one solution when c is zero and a is any real number.

c. The equation will have no solutions when both a and c are positive or both a and c are negative.

36. The graphs of $y = x^2$ and $y = 9$ intersect at $(-3, 9)$ and $(3, 9)$.

$$\begin{aligned} y &= x^2 \\ y &= 9 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

The solutions are $x = 3$ and $x = -3$.

Fair Game Review

37. $(x + 5)^2 = x^2 + 2(x)(5) + 5^2 = x^2 + 10x + 25$

38. $$\begin{aligned} (w - 7)^2 &= w^2 + 2(w)(-7) + (-7^2) \\ &= w^2 - 14w + 49 \end{aligned}$$

39. $$\begin{aligned} (2y - 3)^2 &= (2y)^2 + 2(2y)(-3) + (-3)^2 \\ &= 4y^2 - 12y + 9 \end{aligned}$$

40. B; $a_1 = -3, a_n = a_{n-1} + 2$

$$\begin{aligned} a_n &= a_1 + (n - 1)d \\ a_n &= -3 + (n - 1)(2) \\ a_n &= -3 + 2n - 2 \\ a_n &= 2n - 5 \end{aligned}$$

Chapter 9

Section 9.3

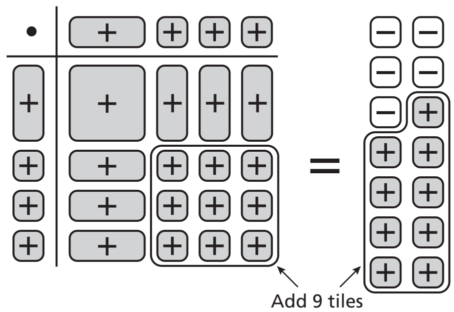
9.3 Activity (pp. 468–469)

1. **Check:** $x = -2 - \sqrt{2}$

$$x^2 + 4x = -2$$
$$\left(-2 - \sqrt{2}\right)^2 + 4\left(-2 - \sqrt{2}\right) \stackrel{?}{=} -2$$
$$4 + 4\sqrt{2} + 2 - 8 - 4\sqrt{2} \stackrel{?}{=} -2$$
$$-2 = -2 \checkmark$$

2. The equation modeled by the algebra tiles is $x^2 + 6x = -5.$

Add 9 tiles to each side.



$$x^2 + 6x + 9 = 4$$
$$(x + 3)^2 = 4$$
$$x + 3 = \pm 2$$
$$\underline{-3} \quad \underline{-3}$$
$$x = -3 \pm 2$$

Check: The solutions are $x = -3 + 2 = -1$ and $x = -3 - 2 = -5.$

$x = -1$:

$$x^2 + 6x = -5$$
$$(-1)^2 + 6(-1) \stackrel{?}{=} -5$$
$$1 - 6 \stackrel{?}{=} -5$$
$$-5 = -5 \checkmark$$

$x = -5$:

$$x^2 + 6x = -5$$
$$(-5)^2 + 6(-5) \stackrel{?}{=} -5$$
$$25 - 30 \stackrel{?}{=} -5$$
$$-5 = -5 \checkmark$$

3. • The group of tiles represents $3x$.
   • The coefficient of x for this group of tiles is half of the coefficient of x in the equation from Activity 2. The number of tiles you add to each side when completing the square is the square of the coefficient of x for this group of tiles.
   • To complete the square take *half* of the coefficient of the x-term and *square* it. *Add* this number to each side of the equation.

4. First, get all the x- and x^2-terms on the left-hand side and the constant on the right-hand side of the equation. Then, to complete the square, take half of the coefficient of the x-term and square it. Add this number to each side of the equation. Then factor the left-hand side as the square of a binomial and evaluate the addition on the right-hand side of the equation. Finally, take the square root of each side and solve for x.

5. **a.** $x^2 - 2x = 1$

$b = -2$

$\dfrac{b}{2} = \dfrac{-2}{2} = -1$

$(-1)^2 = 1$

$x^2 - 2x + 1 = 1 + 1$

$x^2 - 2x + 1 = 2$

$(x - 1)^2 = 2$

$x - 1 = \pm\sqrt{2}$

$x = 1 \pm \sqrt{2}$

The solutions are $x = 1 + \sqrt{2} \approx 2.414$ and $x = 1 - \sqrt{2} \approx -0.414.$

b. $x^2 - 4x = -1$

$b = -4$

$\dfrac{b}{2} = \dfrac{-4}{2} = -2$

$(-2)^2 = 4$

$x^2 - 4x + 4 = -1 + 4$

$x^2 - 4x + 4 = 3$

$(x - 2)^2 = 3$

$x - 2 = \pm\sqrt{3}$

$x = 2 \pm \sqrt{3}$

The solutions are $x = 2 + \sqrt{3} \approx 3.732$ and $x = 2 - \sqrt{3} \approx 0.268.$

c. $x^2 + 4x = -3$

$b = 4$

$\dfrac{b}{2} = \dfrac{4}{2} = 2$

$2^2 = 4$

$x^2 + 4x + 4 = -3 + 4$

$x^2 + 4x + 4 = 1$

$(x + 2)^2 = 1$

$x + 2 = \pm 1$

$x = -2 \pm 1$

The solutions are $x = -2 + 1 = -1$ and $x = -2 - 1 = -3.$

Chapter 9

9.3 On Your Own (pp. 470–471)

1. $x^2 + 10x$

$\frac{b}{2} = \frac{10}{2} = 5$

$5^2 = 25$

$x^2 + 10x + 25 = (x + 5)^2$

2. $x^2 - 4x$

$\frac{b}{2} = \frac{-4}{2} = -2$

$(-2)^2 = 4$

$x^2 - 4x + 4 = (x - 2)^2$

3. $x^2 + 7x$

$\frac{b}{2} = \frac{7}{2}$

$\left(\frac{7}{2}\right)^2 = \frac{49}{4}$

$x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$

4. $x^2 - 6x = 27$

$\frac{b}{2} = \frac{-6}{2} = -3$

$(-3)^2 = 9$

$$\begin{aligned} x^2 - 6x + 9 &= 27 + 9 \\ (x - 3)^2 &= 36 \\ x - 3 &= \pm 6 \\ x &= 3 \pm 6 \end{aligned}$$

The solutions are $x = 3 + 6 = 9$ and $x = 3 - 6 = -3$.

5. $x^2 + 12x + 3 = 1$

$x^2 + 12x = -2$

$\frac{b}{2} = \frac{12}{2} = 6$

$6^2 = 36$

$$\begin{aligned} x^2 + 12x + 36 &= -2 + 36 \\ (x + 6)^2 &= 34 \\ x + 6 &= \pm\sqrt{34} \\ x &= -6 \pm\sqrt{34} \end{aligned}$$

The solutions are $x = -6 + \sqrt{34} \approx -0.169$ and $x = -6 - \sqrt{34} \approx -11.831$.

6. $2x^2 + 4x + 10 = 58$

$2x^2 + 4x = 48$

$x^2 + 2x = 24$

$\frac{b}{2} = \frac{2}{2} = 1$

$1^2 = 1$

$$\begin{aligned} x^2 + 2x + 1 &= 24 + 1 \\ (x + 1)^2 &= 25 \\ x + 1 &= \pm 5 \\ x &= -1 \pm 5 \end{aligned}$$

The solutions are $x = -1 + 5 = 4$ and $x = -1 - 5 = -6$.

7. $h = 0$

$$\begin{aligned} -16t^2 + 64t + 16 &= h \\ -16t^2 + 64t + 16 &= 0 \\ t^2 - 4t - 1 &= 0 \\ t^2 - 4t &= 1 \end{aligned}$$

$\frac{b}{2} = \frac{-4}{2} = -2$

$(-2)^2 = 4$

$$\begin{aligned} t^2 - 4t + 4 &= 1 + 4 \\ (t - 2)^2 &= 5 \\ t - 2 &= \pm\sqrt{5} \\ t &= 2 \pm \sqrt{5} \end{aligned}$$

The solutions are $t = 2 + \sqrt{5} \approx 4.236$ and $t = 2 - \sqrt{5} \approx -0.236$. Use the positive solution.

The stone lands in the water after about 4.2 seconds.

9.3 Exercises (pp. 472–473)

Vocabulary and Concept Check

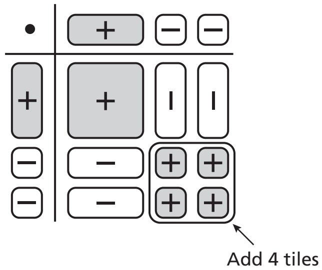
1. To complete the square for an expression of the form $x^2 + bx$, find one-half of b. Square the result and add it to $x^2 + bx$.

2. It is easier to complete the square for $x^2 + bx$ when b is even. Taking half of an even number always results in a whole number while taking half of an odd number always results in a fraction. Squaring a whole number is easier than squaring a fraction.

Chapter 9

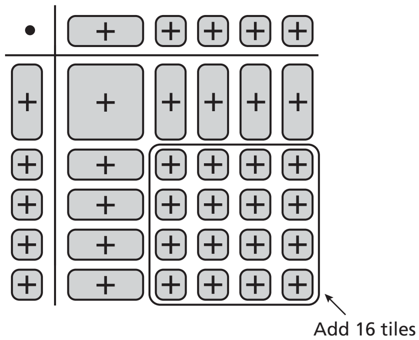
Practice and Problem Solving

3.



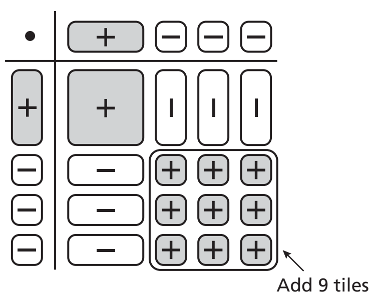
$x^2 - 4x + 4$

4.



$x^2 + 8x + 16$

5.



$x^2 - 6x + 9$

6. $x^2 - 8x + c$

$\frac{b}{2} = \frac{-8}{2} = -4$

$(-4)^2 = 16$

$c = 16$

7. $x^2 + 4x + c$

$\frac{b}{2} = \frac{4}{2} = 2$

$2^2 = 4$

$c = 4$

8. $x^2 - 2x + c$

$\frac{b}{2} = \frac{-2}{2} = -1$

$(-1)^2 = 1$

$c = 1$

9. $x^2 - 14x + c$

$\frac{b}{2} = \frac{-14}{2} = -7$

$(-7)^2 = 49$

$c = 49$

10. $x^2 + 12x + c$

$\frac{b}{2} = \frac{12}{2} = 6$

$6^2 = 36$

$c = 36$

11. $x^2 + 18x + c$

$\frac{b}{2} = \frac{18}{2} = 9$

$9^2 = 81$

$c = 81$

12. $x^2 - 10x$

$\frac{b}{2} = \frac{-10}{2} = -5$

$(-5)^2 = 25$

$x^2 - 10x + 25 = (x - 5)^2$

13. $x^2 + 16x$

$\frac{b}{2} = \frac{16}{2} = 8$

$8^2 = 64$

$x^2 + 16x + 64 = (x + 8)^2$

14. $x^2 + 22x$

$\frac{b}{2} = \frac{22}{2} = 11$

$11^2 = 121$

$x^2 + 22x + 121 = (x + 11)^2$

15. $x^2 - 40x$

$\frac{b}{2} = \frac{-40}{2} = -20$

$(-20)^2 = 400$

$x^2 - 40x + 400 = (x - 20)^2$

16. $x^2 - 3x$

$\frac{b}{2} = \frac{-3}{2}$

$\left(-\frac{3}{2}\right)^2 = \frac{9}{4}$

$x^2 - 3x + \frac{9}{4} = \left(x - \frac{3}{2}\right)^2$

17. $x^2 + 5x$

$\frac{b}{2} = \frac{5}{2}$

$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$x^2 + 5x + \frac{25}{4} = \left(x + \frac{5}{2}\right)^2$

Chapter 9

18. $x^2 + 2x = 3$

$$\frac{b}{2} = \frac{2}{2} = 1$$

$$1^2 = 1$$

$$x^2 + 2x + 1 = 3 + 1$$

$$(x + 1)^2 = 4$$

$$x + 1 = \pm 2$$

$$x = -1 \pm 2$$

The solutions are $x = -1 + 2 = 1$ and $x = -1 - 2 = -3$.

19. $x^2 - 6x = 16$

$$\frac{b}{2} = \frac{-6}{2} = -3$$

$$(-3)^2 = 9$$

$$x^2 - 6x + 9 = 16 + 9$$

$$(x - 3)^2 = 25$$

$$x - 3 = \pm 5$$

$$x = 3 \pm 5$$

The solutions are $x = 3 + 5 = 8$ and $x = 3 - 5 = -2$.

20. $x^2 + 4x + 7 = -6$

$$x^2 + 4x = -13$$

$$\frac{b}{2} = \frac{4}{2} = 2$$

$$2^2 = 4$$

$$x^2 + 4x + 4 = -13 + 4$$

$$(x + 2)^2 = -9$$

There are no real solutions.

21. $x^2 + 5x - 7 = -14$

$$x^2 + 5x = -7$$

$$\frac{b}{2} = \frac{5}{2}$$

$$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

$$x^2 + 5x + \frac{25}{4} = -7 + \frac{25}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = -\frac{3}{4}$$

There are no real solutions.

22. $2x^2 - 8x = 10$

$$x^2 - 4x = 5$$

$$\frac{b}{2} = \frac{-4}{2} = -2$$

$$(-2)^2 = 4$$

$$x^2 - 4x + 4 = 5 + 4$$

$$(x - 2)^2 = 9$$

$$x - 2 = \pm 3$$

$$x = 2 \pm 3$$

The solutions are $x = 2 + 3 = 5$ and $x = 2 - 3 = -1$.

23. $2x^2 - 3x + 1 = 0$

$$2x^2 - 3x = -1$$

$$x^2 - \frac{3}{2}x = -\frac{1}{2}$$

$$\frac{b}{2} = \frac{-\frac{3}{2}}{2} = -\frac{3}{4}$$

$$\left(-\frac{3}{4}\right)^2 = \frac{9}{16}$$

$$x^2 - \frac{3}{2}x + \frac{9}{16} = -\frac{1}{2} + \frac{9}{16}$$

$$\left(x - \frac{3}{4}\right)^2 = \frac{1}{16}$$

$$x - \frac{3}{4} = \pm\frac{1}{4}$$

$$x = \frac{3}{4} \pm \frac{1}{4}$$

The solutions are $x = \frac{3}{4} + \frac{1}{4} = 1$ and $x = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$.

24. The solver did not add 16 to both sides of the equation.

$$x^2 + 8x = 10$$

$$x^2 + 8x + 16 = 10 + 16$$

$$(x + 4)^2 = 26$$

$$x + 4 = \pm\sqrt{26}$$

$$x = -4 \pm\sqrt{26}$$

Chapter 9

25. a. $A = 216, \ell = x + 6, w = x$

$A = \ell w$

$216 = (x + 6)(x)$

$216 = x^2 + 6x$

b. $x^2 + 6x = 216$

$\frac{b}{2} = \frac{6}{2} = 3$

$3^2 = 9$

$x^2 + 6x + 9 = 216 + 9$

$(x + 3)^2 = 225$

$x + 3 = \pm 15$

$x = -3 \pm 15$

The solutions are $x = -3 + 15 = 12$ and $x = -3 - 15 = -18$. Use the positive solution.

The width is 12 feet and the length is $12 + 6 = 18$ feet.

26. $x^2 + bx + 25$

$\left(\frac{b}{2}\right)^2 = 25$

$\frac{b}{2} = \pm 5$

$b = \pm 10$

27. $3x^2 + 6x = 12$

The first step is to divide each side by 3 so that the coefficient of x^2 is 1.

28. a. $x^2 + 4x - 12 = 0$

$x^2 + 4x = 12$

$\frac{b}{2} = \frac{4}{2} = 2$

$2^2 = 4$

$x^2 + 4x + 4 = 12 + 4$

$(x + 2)^2 = 16$

$x + 2 = \pm 4$

$x = -2 \pm 4$

The solutions are $x = -2 + 4 = 2$ and $x = -2 - 4 = -6$.

b. To find the minimum value, evaluate $y = x^2 + 4x - 12$ when x is the mean of the solutions.

$x = \frac{2 + (-6)}{2} = \frac{-4}{2} = -2$

$y = x^2 + 4x - 12$

$= (-2)^2 + 4(-2) - 12$

$= 4 - 8 - 12$

$= -16$

29. a. $h = -16t^2 + 64t + 32; h = 0$

$-16t^2 + 64t + 32 = 0$

$-16t^2 + 64t = -32$

$t^2 - 4t = 2$

$\frac{b}{2} = \frac{-4}{2} = -2$

$(-2)^2 = 4$

$t^2 - 4t + 4 = 2 + 4$

$(t - 2)^2 = 6$

$t - 2 = \pm\sqrt{6}$

$t = 2 \pm \sqrt{6}$

The solutions are $t = 2 + \sqrt{6} \approx 4.4$ and $t = 2 - \sqrt{6} \approx -0.4$. Use the positive solution.

The rocket hits the ground after about 4.4 seconds.

b. The maximum of the function occurs when t is the mean of the zeros. So, the maximum of $h = -16t^2 + 64t + 32$ occurs when

$t = \frac{4.4 + (-0.4)}{2} = 2.$

$h = -16t^2 + 64t + 32$

$h = -16(2)^2 + 64(2) + 32$

$h = -16(4) + 128 + 32$

$h = -64 + 128 + 32$

$h = 96$

The maximum height of the rocket is 96 feet.

30. A = area, P = perimeter, ℓ = length, w = width

$A = \ell w = 100, P = 2\ell + w = 40$

$$\begin{aligned} 40 &= 2\ell + w \\ \underline{-2\ell} &\quad \underline{-2\ell} \\ 40 - 2\ell &= w \end{aligned}$$

$$\begin{aligned} \ell w &= 100 \\ \ell(40 - 2\ell) &= 100 \\ 40\ell - 2\ell^2 &= 100 \\ -2\ell^2 + 40\ell &= 100 \\ \ell^2 - 20\ell &= -50 \end{aligned}$$

$\frac{b}{2} = \frac{-20}{2} = -10$

$(-10)^2 = 100$

$$\begin{aligned} \ell^2 - 20\ell + 100 &= -50 + 100 \\ (\ell - 10)^2 &= 50 \\ \ell - 10 &= \pm 5\sqrt{2} \\ \ell &= 10 \pm 5\sqrt{2} \end{aligned}$$

The length of the garden is $\ell = 10 + 5\sqrt{2} \approx 17.1$ feet or $\ell = 10 - 5\sqrt{2} \approx 2.9$ feet.

The width is $w = 40 - 2(17.1) = 5.8$ feet or $w = 40 - 2(2.9) = 34.2$ feet.

The best choice is to make the garden 17.1 feet long and 5.8 feet wide because the dimensions give a wider rectangle.

31. x = the first positive integer, y = the second positive integer

$$\begin{aligned} x + 1 &= y \\ xy &= 42 \\ x(x + 1) &= 42 \\ x^2 + x &= 42 \end{aligned}$$

$\frac{b}{2} = 0.5$

$(0.5)^2 = 0.25$

$$\begin{aligned} x^2 + x + 0.25 &= 42 + 0.25 \\ (x + 0.5)^2 &= 42.25 \\ x + 0.5 &= \pm 6.5 \\ x &= -0.5 \pm 6.5 \end{aligned}$$

The solutions are $x = -0.5 + 6.5 = 6$ and $x = -0.5 - 6.5 = -7$. Use the positive solution.

$$\begin{aligned} x + 1 &= y \\ 6 + 1 &= y \\ 7 &= y \end{aligned}$$

The integers are 6 and 7.

32.
$$\begin{aligned} x^2 + 4x + 3 &= 0 \\ x^2 + 4x &= -3 \\ x^2 + 4x + 4 &= -3 + 4 \\ (x + 2)^2 &= 1 \end{aligned}$$

a. $y = (x + 2)^2 - 1$

Vertex: $(-2, -1)$

The parabola opens up, so -1 is the minimum value.

b. $y = x^2 + bx + c$

$$\begin{aligned} x^2 + bx + c &= 0 \\ x^2 + bx &= -c \\ x^2 + bx + \left(\frac{b}{2}\right)^2 &= -c + \left(\frac{b}{2}\right)^2 \\ \left(x + \frac{b}{2}\right)^2 &= -c + \frac{b^2}{4} \\ \left(x + \frac{b}{2}\right)^2 + c - \frac{b^2}{4} &= 0 \\ y &= \left(x + \frac{b}{2}\right)^2 + c - \frac{b^2}{4} \end{aligned}$$

The minimum value is $c - \frac{b^2}{4}$.

Fair Game Review

33. $a = 3, b = -6, c = 2$

$$\begin{aligned} \sqrt{b^2 - 4ac} &= \sqrt{(-6)^2 - 4(3)(2)} \\ &= \sqrt{36 - 24} \\ &= \sqrt{12} \\ &= \sqrt{4 \bullet 3} \\ &= \sqrt{4} \bullet \sqrt{3} \\ &= 2\sqrt{3} \end{aligned}$$

34. $a = -2, b = 4, c = 7$

$$\begin{aligned} \sqrt{b^2 - 4ac} &= \sqrt{4^2 - 4(-2)(7)} \\ &= \sqrt{16 + 56} \\ &= \sqrt{72} \\ &= \sqrt{36 \bullet 2} \\ &= \sqrt{36} \bullet \sqrt{2} \\ &= 6\sqrt{2} \end{aligned}$$

35. $a = 1, b = 6, c = 4$

$$\begin{aligned}\sqrt{b^2 - 4ac} &= \sqrt{6^2 - 4(1)(4)} \\ &= \sqrt{36 - 16} \\ &= \sqrt{20} \\ &= \sqrt{4 \bullet 5} \\ &= \sqrt{4} \bullet \sqrt{5} \\ &= 2\sqrt{5}\end{aligned}$$

36. B;

$$\begin{aligned}x^2 - 49 &= 0 \\ \underline{+49} &\quad \underline{+49} \\ x^2 &= 49 \\ \sqrt{x^2} &= \sqrt{49} \\ x &= \pm 7\end{aligned}$$

The solutions are $x = 7$ and $x = -7$.

Study Help (p. 474)

Available at *BigIdeasMath.com*.

Quiz 9.1–9.3 (p. 475)

1. $x^2 - 2x - 3 = 0$

The solutions are $x = -1$ and $x = 3$.

Check:

$x = -1$:
$$\begin{aligned}x^2 - 2x - 3 &= 0 \\ (-1)^2 - 2(-1) - 3 &\stackrel{?}{=} 0 \\ 1 + 2 - 3 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark\end{aligned}$$

$x = 3$:
$$\begin{aligned}x^2 - 2x - 3 &= 0 \\ 3^2 - 2(3) - 3 &\stackrel{?}{=} 0 \\ 9 - 6 - 3 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark\end{aligned}$$

2. $x^2 - 2x + 3 = 0$

The graph does not intersect the x-axis. So, $x^2 - 2x + 3 = 0$ has no real solutions.

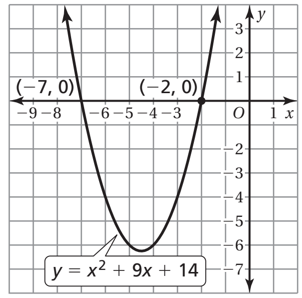
3. $x^2 + 10x + 25 = 0$

The solution is $x = -5$.

Check:

$x = -5$:
$$\begin{aligned}x^2 + 10x + 25 &= 0 \\ (-5)^2 + 10(-5) + 25 &\stackrel{?}{=} 0 \\ 25 - 50 + 25 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark\end{aligned}$$

4. $x^2 + 9x + 14 = 0$



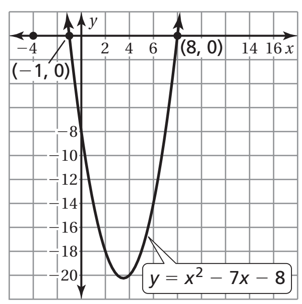
The solutions are $x = -7$ and $x = -2$.

Check:

$x = -7$:
$$\begin{aligned}x^2 + 9x + 14 &= 0 \\ (-7)^2 + 9(-7) + 14 &\stackrel{?}{=} 0 \\ 49 - 63 + 14 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark\end{aligned}$$

$x = -2$:
$$\begin{aligned}x^2 + 9x + 14 &= 0 \\ (-2)^2 + 9(-2) + 14 &\stackrel{?}{=} 0 \\ 4 - 18 + 14 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark\end{aligned}$$

5.
$$\begin{aligned}x^2 - 7x &= 8 \\ x^2 - 7x - 8 &= 0\end{aligned}$$



The solutions are $x = -1$ and $x = 8$.

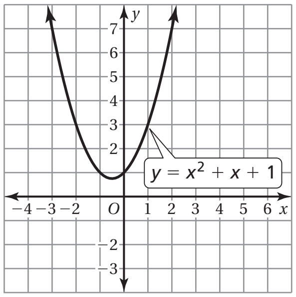
$x = -1$:
$$\begin{aligned}x^2 - 7x &= 8 \\ (-1)^2 - 7(-1) &\stackrel{?}{=} 8 \\ 1 + 7 &\stackrel{?}{=} 8 \\ 8 &= 8 \checkmark\end{aligned}$$

$x = 8$:
$$\begin{aligned}x^2 - 7x &= 8 \\ 8^2 - 7(8) &\stackrel{?}{=} 8 \\ 64 - 56 &\stackrel{?}{=} 8 \\ 8 &= 8 \checkmark\end{aligned}$$

Chapter 9

6. $x + 1 = -x^2$

$x^2 + x + 1 = 0$



The graph does not intersect the x-axis. So, $x + 1 = -x^2$ has no real solutions.

7. $4x^2 = 64$

$\frac{4x^2}{4} = \frac{64}{4}$

$x^2 = 16$

$x = \pm\sqrt{16}$

$x = \pm 4$

The solutions are $x = 4$ and $x = -4$.

8. $-3x^2 + 6 = 10$

$\underline{-6} \quad \underline{-6}$

$-3x^2 = 4$

$\frac{-3x^2}{-3} = \frac{4}{-3}$

$x^2 = -\frac{4}{3}$

The equation has no real solutions.

9. $(x - 8)^2 = 1$

$x - 8 = \pm 1$

$\underline{+8} \quad \underline{+8}$

$x = 8 \pm 1$

The solutions are $x = 8 + 1 = 9$ and $x = 8 - 1 = 7$.

10. $x^2 + 4x = 45$

$\frac{b}{2} = \frac{4}{2} = 2$

$2^2 = 4$

$x^2 + 4x + 4 = 45 + 4$

$(x + 2)^2 = 49$

$x + 2 = \pm 7$

$x = -2 \pm 7$

The solutions are $x = -2 + 7 = 5$ and $x = -2 - 7 = -9$.

11. $x^2 - 2x - 1 = 8$

$x^2 - 2x = 9$

$\frac{b}{2} = \frac{-2}{2} = -1$

$(-1)^2 = 1$

$x^2 - 2x + 1 = 9 + 1$

$(x - 1)^2 = 10$

$x - 1 = \pm\sqrt{10}$

$x = 1 \pm \sqrt{10}$

The solutions are $x = 1 + \sqrt{10} \approx 4.162$ and $x = 1 - \sqrt{10} \approx -2.162$.

12. $2x^2 + 12x + 20 = 34$

$2x^2 + 12x = 14$

$x^2 + 6x = 7$

$\frac{b}{2} = \frac{6}{2} = 3$

$3^2 = 9$

$x^2 + 6x + 9 = 7 + 9$

$(x + 3)^2 = 16$

$x + 3 = \pm 4$

$x = -3 \pm 4$

The solutions are $x = -3 + 4 = 1$ and $x = -3 - 4 = -7$.

13. $-4x^2 + 8x + 44 = 16$

$-4x^2 + 8x = -28$

$x^2 - 2x = 7$

$\frac{b}{2} = \frac{-2}{2} = -1$

$(-1)^2 = 1$

$x^2 - 2x + 1 = 7 + 1$

$(x - 1)^2 = 8$

$x - 1 = \pm 2\sqrt{2}$

$x = 1 \pm 2\sqrt{2}$

The solutions are $x = 1 + 2\sqrt{2} \approx 3.828$ and $x = 1 - 2\sqrt{2} \approx -1.828$.

14. $x^2 = 100$

The equation $x^2 = 100$ is of the form $x^2 = d$, where d is positive. When d is positive, the equation has two real solutions.

Chapter 9

15. $V = 380, \ell = 4w, h = 5$

$$\begin{aligned} V &= \ell wh \\ 380 &= (4w)(w)(5) \\ 380 &= 20w^2 \\ \frac{380}{20} &= \frac{20w^2}{20} \\ 19 &= w^2 \\ \pm\sqrt{19} &= w \end{aligned}$$

The solutions are $w = \sqrt{19} \approx 4.36$ and $w = -\sqrt{19} \approx -4.36$. Use the positive solution.

So, the width is about 4.36 meters and the length is about $4(4.36) = 17.44$ meters.

16. a. $h = 10$ ft

$$\begin{aligned} h &= -16t^2 + 40t + 3 \\ 10 &= -16t^2 + 40t + 3 \\ 7 &= -16t^2 + 40t \\ -\frac{7}{16} &= t^2 - \frac{5}{2}t \end{aligned}$$

$$\frac{b}{2} = \frac{\frac{-5}{2}}{2} = -\frac{5}{4}$$

$$\left(-\frac{5}{4}\right)^2 = \frac{25}{16}$$

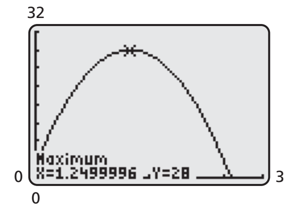
$$\begin{aligned} t^2 - \frac{5}{2}t + \frac{25}{16} &= -\frac{7}{16} + \frac{25}{16} \\ \left(t - \frac{5}{4}\right)^2 &= \frac{9}{8} \\ t - \frac{5}{4} &= \pm\sqrt{\frac{9}{8}} \\ t &= \frac{5}{4} \pm \sqrt{\frac{9}{8}} \end{aligned}$$

The solutions are $t = \frac{5}{4} + \sqrt{\frac{9}{8}} \approx 2.31$ and $t = \frac{5}{4} - \sqrt{\frac{9}{8}} \approx 0.19$.

So, the cannonball is 10 feet above the ground after about 0.19 second and about 2.31 seconds.

b. $h = -16t^2 + 40t + 3$

$y = -16x^2 + 40x + 3$



The maximum is at $(1.25, 28)$. So, the maximum height of the cannonball is 28 feet.

Section 9.4

9.4 Activity (pp. 476–477)

1. 1. $ax^2 + bx + c = 0$

2. $4a^2x^2 + 4abx + 4ac = 0$

Each side of the equation was multiplied by $4a$.

3. $4a^2x^2 + 4abx + 4ac + b^2 = b^2$

The term b^2 was added to each side of the equation.

4. $4a^2x^2 + 4abx + b^2 = b^2 - 4ac$

The term $4ac$ was subtracted from each side of the equation.

5. $(2ax + b)^2 = b^2 - 4ac$

The left-hand side of the equation was of the form $ax^2 + bx + c$ and is a perfect square trinomial. It was factored.

6. $2ax + b = \pm\sqrt{b^2 - 4ac}$

The square root of each side of the equation was taken.

7. $2ax = -b \pm \sqrt{b^2 - 4ac}$

The term b was subtracted from each side of the equation.

8. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Each side of the equation was divided by the term $2a$.

Chapter 9

2.
$$\begin{aligned} ax^2 + bx + c &= 0 \\ -c \quad &\quad -c \\ ax^2 + bx &= -c \\ \frac{ax^2 + bx}{a} &= \frac{-c}{a} \\ x^2 + \frac{b}{a}x &= -\frac{c}{a} \end{aligned}$$

$$\frac{b}{2} = \frac{\frac{b}{a}}{2} = \frac{b}{2a}$$

$$\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$\begin{aligned} x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} &= -\frac{c}{a} + \frac{b^2}{4a^2} \\ \left(x + \frac{b}{2a}\right)^2 &= -\frac{c}{a} + \frac{b^2}{4a^2} \\ \left(x + \frac{b}{2a}\right)^2 &= -\frac{4ac}{4a^2} + \frac{b^2}{4a^2} \\ \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\ x + \frac{b}{2a} &= \frac{\pm\sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned}$$

The second method is longer than the first, but both can be represented by the quadratic formula. The terms $4a$ and b^2 in steps 2 and 3 of Activity 1 make the equation a perfect square trinomial, allowing it to be factored.

3. a. 1 rational solution

The discriminant is equal to 0 because the graph intercepts the x-axis at one point.

b. 2 rational solutions

The discriminant is greater than 0 because the graph has two x-intercepts.

c. 2 irrational solutions

The discriminant is greater than 0 because the graph has two x-intercepts.

d. no real solutions

The discriminant is less than 0 because the graph has no x-intercepts.

4. If the discriminant is greater than 0, the graph has two x-intercepts and two real solutions. If the discriminant is equal to 0, the graph has one x-intercept and one real solution. If the discriminant is less than 0, the graph has no x-intercepts and no real solutions.

5. a. $x^2 + 2x - 3 = 0$

$a = 1, b = 2, c = -3$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{2^2 - 4(1)(-3)}}{2(1)} \\ &= \frac{-2 \pm \sqrt{16}}{2} \\ &= \frac{-2 \pm 4}{2} \\ &= -1 \pm 2 \end{aligned}$$

So, the solutions are $x = -1 + 2 = 1$ and $x = -1 - 2 = -3$.

b. $x^2 - 4x + 4 = 0$

$a = 1, b = -4, c = 4$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)} \\ &= \frac{4 \pm 0}{2} \\ &= \frac{4}{2} \\ &= 2 \end{aligned}$$

The solution is $x = 2$.

c. $x^2 + 4x + 5 = 0$

$a = 1, b = 4, c = 5$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)} \\ &= \frac{-4 \pm \sqrt{-4}}{2} \end{aligned}$$

The equation has no real solutions.

6. An imaginary number is a number whose square is negative. Quadratic equations have solutions containing imaginary numbers when they have a negative discriminant.

Chapter 9

9.4 On Your Own (pp. 478–480)

1. $x^2 - 6x + 5 = 0$

$a = 1, b = -6, c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$$
$$= \frac{6 \pm \sqrt{16}}{2}$$
$$= \frac{6 \pm 4}{2}$$
$$= 3 \pm 2$$

So, the solutions are $x = 3 + 2 = 5$ and $x = 3 - 2 = 1$.

Check:

$x = 5$:

$$x^2 - 6x + 5 = 0$$
$$5^2 - 6(5) + 5 \stackrel{?}{=} 0$$
$$25 - 30 + 5 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

$x = 1$:

$$x^2 - 6x + 5 = 0$$
$$1^2 - 6(1) + 5 \stackrel{?}{=} 0$$
$$1 - 6 + 5 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

2. $4x^2 + x - 3 = 0$

$a = 4, b = 1, c = -3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1^2 - 4(4)(-3)}}{2(4)}$$
$$= \frac{-1 \pm \sqrt{49}}{8}$$
$$= \frac{-1 \pm 7}{8}$$

So, the solutions are $x = \frac{-1 + 7}{8} = \frac{3}{4}$ and $x = \frac{-1 - 7}{8} = -1$.

Check:

$x = \frac{3}{4}$:

$$4x^2 + x - 3 = 0$$
$$4\left(\frac{3}{4}\right)^2 + \frac{3}{4} - 3 \stackrel{?}{=} 0$$
$$4\left(\frac{9}{16}\right) - \frac{9}{4} \stackrel{?}{=} 0$$
$$\frac{9}{4} - \frac{9}{4} \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

$x = -1$:

$$4x^2 + x - 3 = 0$$
$$4(-1)^2 + (-1) - 3 \stackrel{?}{=} 0$$
$$4(1) - 4 \stackrel{?}{=} 0$$
$$4 - 4 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

3. $-6x^2 + 7x - 2 = 0$

$a = -6, b = 7, c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-7 \pm \sqrt{7^2 - 4(-6)(-2)}}{2(-6)}$$
$$= \frac{-7 \pm \sqrt{1}}{-12}$$
$$= \frac{-7 \pm 1}{-12}$$

So, the solutions are $x = \frac{-7 + 1}{-12} = \frac{1}{2}$ and $x = \frac{-7 - 1}{-12} = \frac{2}{3}$.

Check:

$x = \frac{1}{2}$:

$$-6x^2 + 7x - 2 = 0$$
$$-6\left(\frac{1}{2}\right)^2 + 7\left(\frac{1}{2}\right) - 2 \stackrel{?}{=} 0$$
$$-6\left(\frac{1}{4}\right) + \frac{7}{2} - 2 \stackrel{?}{=} 0$$
$$-\frac{3}{2} + \frac{7}{2} - 2 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

$x = \frac{2}{3}$:

$$-6x^2 + 7x - 2 = 0$$
$$-6\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) - 2 \stackrel{?}{=} 0$$
$$-6\left(\frac{4}{9}\right) + \frac{14}{3} - 2 \stackrel{?}{=} 0$$
$$-\frac{8}{3} + \frac{14}{3} - 2 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

4. $4x^2 - 4x + 1 = 0$

$a = 4, b = -4, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1)}}{2(4)}$$
$$= \frac{4 \pm \sqrt{0}}{8}$$
$$= \frac{4}{8}$$
$$= \frac{1}{2}$$

The solution is $x = \frac{1}{2}$.

Chapter 9

5. $-5x^2 + x = -4$

$-5x^2 + x + 4 = 0$

$a = -5, b = 1, c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1^2 - 4(-5)(4)}}{2(-5)}$$
$$= \frac{-1 \pm \sqrt{81}}{-10}$$
$$= \frac{-1 \pm 9}{-10}$$

The solutions are $x = \frac{-1 + 9}{-10} = -\frac{4}{5}$ and

$x = \frac{-1 - 9}{-10} = 1.$

6. $3x^2 + 2x = 5$

$3x^2 + 2x - 5 = 0$

$a = 3, b = 2, c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{2^2 - 4(3)(-5)}}{2(3)}$$
$$= \frac{-2 \pm \sqrt{64}}{6}$$
$$= \frac{-2 \pm 8}{6}$$
$$= \frac{-1 \pm 4}{3}$$

The solutions are $x = \frac{-1 + 4}{3} = 1$ and

$x = \frac{-1 - 4}{3} = -\frac{5}{3}.$

7. $85 = 0.34x^2 + 3.0x + 9$

$0 = 0.34x^2 + 3.0x - 76$

$a = 0.34, b = 3.0, c = -76$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3.0 \pm \sqrt{(3.0)^2 - 4(0.34)(-76)}}{2(0.34)}$$
$$= \frac{-3.0 \pm \sqrt{112.36}}{0.68}$$

The solutions are $x = \frac{-3.0 + \sqrt{112.36}}{0.68} \approx 11$ and

$x = \frac{-3 - \sqrt{112.36}}{0.68} \approx -20.$

Because x represents the number of years since 1995, x is greater than or equal to zero. So, there were about 85 breeding pairs 11 years after 1995, in 2006.

8. $-x^2 + 4x - 4 = 0$

$a = -1, b = 4, c = -4$

$b^2 - 4ac = 4^2 - 4(-1)(-4) = 16 - 16 = 0$

The discriminant is 0, so the equation has one real solution.

9. $6x^2 + 2x = -1$

$6x^2 + 2x + 1 = 0$

$a = 6, b = 2, c = 1$

$b^2 - 4ac = 2^2 - 4(6)(1) = 4 - 24 = -20$

The discriminant is less than 0, so the equation has no real solutions.

10. $\frac{1}{2}x^2 = 7x - 1$

$\frac{1}{2}x^2 - 7x + 1 = 0$

$a = \frac{1}{2}, b = -7, c = 1$

$b^2 - 4ac = (-7)^2 - 4\left(\frac{1}{2}\right)(1) = 49 - 2 = 47$

The discriminant is greater than 0, so the equation has two real solutions.

9.4 Exercises (pp. 481–483)

Vocabulary and Concept Check

1. The real solutions of the quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a \neq 0$ and $b^2 - 4ac \geq 0$.

Chapter 9

2. If the discriminant is greater than 0, there are two x-intercepts and two real solutions.

If the discriminant is equal to 0, there is one x-intercept and one real solution.

If the discriminant is less than 0, there are no x-intercepts and no real solutions.

Practice and Problem Solving

3.
$$x^2 = 7x$$
$$x^2 - 7x = 0$$
$a = 1$

$b = -7$

$c = 0$

4.
$$x^2 - 4x = -12$$
$$x^2 - 4x + 12 = 0$$
$a = 1$

$b = -4$

$c = 12$

5.
$$-2x^2 + 1 = 5x$$
$$-2x^2 - 5x + 1 = 0$$
$a = -2$

$b = -5$

$c = 1$

6.
$$3x + 2 = 4x^2$$
$$-4x^2 + 3x + 2 = 0$$
$a = -4$

$b = 3$

$c = 2$

7.
$$4 - 6x = -x^2$$
$$x^2 - 6x + 4 = 0$$
$a = 1$

$b = -6$

$c = 4$

8.
$$-8x = 3x^2 + 3$$
$$0 = 3x^2 + 8x + 3$$
$a = 3$

$b = 8$

$c = 3$

9. $x^2 - 12x + 36 = 0$

$a = 1, b = -12, c = 36$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(36)}}{2(1)} \\ &= \frac{12 \pm \sqrt{0}}{2} \\ &= \frac{12 \pm 0}{2} \\ &= \frac{12}{2} \\ &= 6 \end{aligned}$$

The solution is $x = 6$.

10. $x^2 + 7x + 16 = 0$

$a = 1, b = 7, c = 16$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-7 \pm \sqrt{7^2 - 4(1)(16)}}{2(1)} \\ &= \frac{-7 \pm \sqrt{-15}}{2} \end{aligned}$$

The equation has no real solutions.

11. $x^2 - 10x - 11 = 0$

$a = 1, b = -10, c = -11$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-11)}}{2(1)} \\ &= \frac{10 \pm \sqrt{144}}{2} \\ &= \frac{10 \pm 12}{2} \\ &= 5 \pm 6 \end{aligned}$$

The solutions are $x = 5 + 6 = 11$ and $x = 5 - 6 = -1$.

12. $2x^2 - x - 1 = 0$

$a = 2, b = -1, c = -1$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{1 \pm \sqrt{9}}{4} \\ &= \frac{1 \pm 3}{4} \end{aligned}$$

The solutions are $x = \frac{1 + 3}{4} = 1$ and $x = \frac{1 - 3}{4} = -\frac{1}{2}$.

13. $2x^2 - 6x + 5 = 0$

$a = 2, b = -6, c = 5$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(5)}}{2(2)} \\ &= \frac{6 \pm \sqrt{-4}}{4} \end{aligned}$$

The equation has no real solutions.

14. $9x^2 - 6x + 1 = 0$

$a = 9, b = -6, c = 1$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)} \\ &= \frac{6 \pm \sqrt{0}}{18} \\ &= \frac{6 \pm 0}{18} \\ &= \frac{6}{18} \\ &= \frac{1}{3} \end{aligned}$$

The solution is $x = \frac{1}{3}$.

15. $6x^2 - 13x = -6$

$6x^2 - 13x + 6 = 0$

$a = 6, b = -13, c = 6$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-13) \pm \sqrt{(-13)^2 - 4(6)(6)}}{2(6)} \\ &= \frac{13 \pm \sqrt{25}}{12} \\ &= \frac{13 \pm 5}{12} \end{aligned}$$

The solutions are $x = \frac{13 + 5}{12} = \frac{3}{2}$ and $x = \frac{13 - 5}{12} = \frac{2}{3}$.

16. $-3x^2 + 6x = 4$

$-3x^2 + 6x - 4 = 0$

$a = -3, b = 6, c = -4$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-6 \pm \sqrt{6^2 - 4(-3)(-4)}}{2(-3)} \\ &= \frac{-6 \pm \sqrt{-12}}{-6} \end{aligned}$$

The equation has no real solutions.

17. $1 - 8x = -16x^2$

$16x^2 - 8x + 1 = 0$

$a = 16, b = -8, c = 1$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(16)(1)}}{2(16)} \\ &= \frac{8 \pm \sqrt{0}}{32} \\ &= \frac{8 \pm 0}{32} \\ &= \frac{8}{32} \\ &= \frac{1}{4} \end{aligned}$$

The solution is $x = \frac{1}{4}$.

18. $x^2 - 5x + 3 = 0$

$a = 1, b = -5, c = 3$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)} \\ &= \frac{5 \pm \sqrt{13}}{2} \end{aligned}$$

The solutions are $x = \frac{5 + \sqrt{13}}{2} \approx 4.3$ and $x = \frac{5 - \sqrt{13}}{2} \approx 0.7$.

19. $x^2 + 2x = 9$

$x^2 + 2x - 9 = 0$

$a = 1, b = 2, c = -9$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{2^2 - 4(1)(-9)}}{2(1)} \\ &= \frac{-2 \pm \sqrt{40}}{2} \end{aligned}$$

The solutions are $x = \frac{-2 + \sqrt{40}}{2} \approx 2.2$ and $x = \frac{-2 - \sqrt{40}}{2} \approx -4.2$.

20. $5x^2 - 2 = 4x$

$5x^2 - 4x - 2 = 0$

$a = 5, b = -4, c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(-2)}}{2(5)} = \frac{4 \pm \sqrt{56}}{10} = \frac{2 \pm \sqrt{14}}{5}$$

The solutions are $x = \frac{2 + \sqrt{14}}{5} \approx 1.1$ and $x = \frac{2 - \sqrt{14}}{5} \approx -0.3$.

21. The solver did not change the sign of -7 in front of the discriminant.

$3x^2 - 7x - 6 = 0$

$a = 3, b = -7, c = -6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-6)}}{2(3)} = \frac{7 \pm \sqrt{121}}{6} = \frac{7 \pm 11}{6}$$

The solutions are $x = \frac{7 + 11}{6} = 3$ and $x = \frac{7 - 11}{6} = -\frac{2}{3}$.

22. The solver used 4 for c instead of -4.

$-2x^2 + 9x = 4$

$-2x^2 + 9x - 4 = 0$

$a = -2, b = 9, c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-9 \pm \sqrt{9^2 - 4(-2)(-4)}}{2(-2)} = \frac{-9 \pm \sqrt{49}}{-4} = \frac{-9 \pm 7}{-4}$$

The solutions are $x = \frac{-9 + 7}{-4} = \frac{1}{2}$ and $x = \frac{-9 - 7}{-4} = 4$.

23. $h = -0.1d^2 + 0.1d + 3$

$0 = -0.1d^2 + 0.1d + 3$

$a = -0.1, b = 0.1, c = 3$

$$d = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-0.1 \pm \sqrt{(0.1)^2 - 4(-0.1)(3)}}{2(-0.1)} = \frac{-0.1 \pm \sqrt{1.21}}{-0.2} = \frac{-0.1 \pm 1.1}{-0.2}$$

The solutions are $d = \frac{-0.1 + 1.1}{-0.2} = -5$ and $d = \frac{-0.1 - 1.1}{-0.2} = 6$. Use the positive solution. The swimmer enters the water 6 feet from the pier.

24. C; $b^2 - 4ac > 0$

If the discriminant is greater than 0, there are two x-intercepts and two real solutions.

25. A; $b^2 - 4ac = 0$

If the discriminant is equal to 0, there is one x-intercept and one real solution.

26. B; $b^2 - 4ac < 0$

If the discriminant is less than 0, there are no x-intercepts and no real solutions.

27. $x^2 - 6x - 10 = 0$

$a = 1, b = -6, c = 10$

$b^2 - 4ac = (-6)^2 - 4(1)(10) = 36 - 40 = -4$

The discriminant is less than 0, so the equation has no real solutions.

28. $x^2 - 5x - 3 = 0$

$a = 1, b = -5, c = -3$

$b^2 - 4ac = (-5)^2 - 4(1)(-3) = 25 + 12 = 37$

The discriminant is greater than 0, so the equation has two real solutions.

29. $2x^2 - 12x = -18$

$2x^2 - 12x + 18 = 0$

$a = 2, b = -12, c = 18$

$b^2 - 4ac = (-12)^2 - 4(2)(18) = 144 - 144 = 0$

The discriminant is 0, so the equation has one real solution.

30. $4x^2 = 4x - 1$

$4x^2 - 4x + 1 = 0$

$a = 4, b = -4, c = 1$

$b^2 - 4ac = (-4)^2 - 4(4)(1) = 16 - 16 = 0$

The discriminant is 0, so the equation has one real solution.

31. $-\frac{1}{4}x^2 + 4x = -2$

$-\frac{1}{4}x^2 + 4x + 2 = 0$

$a = -\frac{1}{4}, b = 4, c = 2$

$b^2 - 4ac = 4^2 - 4\left(-\frac{1}{4}\right)(2) = 16 + 2 = 18$

The discriminant is greater than 0, so the equation has two real solutions.

32. $-5x^2 + 8x = 9$

$-5x^2 + 8x - 9 = 0$

$a = -5, b = 8, c = -9$

$b^2 - 4ac = 8^2 - 4(-5)(-9) = 64 - 180 = -116$

The discriminant is less than 0, so the equation has no real solutions.

33. a. Yes, you could have used factoring to solve the equation. Integer solutions indicate that b is the sum of two factors of c.

b. Yes, you could have used factoring to solve the equation. Fractional solutions indicate that the problem can be solved by completing the square.

c. In general, quadratic equations with rational solutions are factorable.

34. $d = 235$

$d = 0.05v^2 + 2.2v$

$235 = 0.05v^2 + 2.2v$

$0 = 0.05v^2 + 2.2v - 235$

$235 = 0.05v^2 + 2.2v$

$0 = 0.05v^2 + 2.2v - 235$

$a = 0.05, b = 2.2, c = -235$

$$v = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2.2 \pm \sqrt{(2.2)^2 - 4(0.05)(-235)}}{2(0.05)}$$

$$= \frac{-2.2 \pm \sqrt{51.84}}{0.1}$$

The solutions are $v = \frac{-2.2 + \sqrt{51.84}}{0.1} = 50$ and

$v = \frac{-2.2 - \sqrt{51.84}}{0.1} = -94$. Use the positive solution.

The car was going 50 miles per hour when the brakes were applied.

35. a. $y = 15$

$y = -0.08x^2 + 1.6x + 10$

$15 = -0.08x^2 + 1.6x + 10$

$0 = -0.08x^2 + 1.6x - 5$

$a = -0.08, b = 1.6, c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1.6 \pm \sqrt{(1.6)^2 - 4(-0.08)(-5)}}{2(-0.08)}$$

$$= \frac{-1.6 \pm \sqrt{0.96}}{-0.16}$$

The solutions are $x = \frac{-1.6 + \sqrt{0.96}}{-0.16} \approx 3.9$ and

$x = \frac{-1.6 - \sqrt{0.96}}{-0.16} \approx 16.1$.

Because x represents the number of years since 1990, there were about 15 tons of trout caught in the lake 4 years after 1990, in 1994, and 16 years after 1990, in 2006.

b. No, this model cannot be used for future years because the model predicts negative numbers of fish caught after 2015.

Chapter 9

36. The solver did not add 11 to each side of the equation prior to using the discriminant.

$$2x^2 - 5x - 2 = -11$$
$$2x^2 - 5x + 9 = 0$$
$$a = 2, b = -5, c = 9$$
$$b^2 - 4ac = (-5)^2 - 4(2)(9) = 25 - 72 = -47$$

The discriminant is less than 0, so the equation has no real solutions.

37. $x^2 + 5x - 1 = 0$

$$a = 1, b = 5, c = -1$$
$$b^2 - 4ac = 5^2 - 4(1)(-1) = 25 + 4 = 29$$

The discriminant is greater than 0, so the graph of the related function intersects the x-axis in two places.

38.
$$4x^2 + 4x = -1$$
$$4x^2 + 4x + 1 = 0$$
$$a = 4, b = 4, c = 1$$
$$b^2 - 4ac = 4^2 - 4(4)(1) = 16 - 16 = 0$$

The discriminant is 0, so the graph of the related function intersects the x-axis in one place.

39.
$$4 - 3x = -6x^2$$
$$6x^2 - 3x + 4 = 0$$
$$a = 6, b = -3, c = 4$$
$$b^2 - 4ac = (-3)^2 - 4(6)(4) = 9 - 96 = -87$$

The discriminant is less than 0, so the graph of the related function does not intersect the x-axis.

40. $x^2 + 3x + c = 0$

a. *Sample answer:* $c = 2$

b. *Sample answer:* $c = -20$

41. $x^2 - 6x + c = 0$

a. *Sample answer:* $c = 8$

b. *Sample answer:* $c = 2$

42. $x^2 - 8x + c = 0$

a. *Sample answer:* $c = 16$

b. *Sample answer:* $c = -7$

43. When a and c have different signs, the graph of the related function has two x-intercepts and therefore always has two real solutions.

44. When the discriminant is a perfect square, the solutions of $ax^2 + bx + c = 0$ are rational because the square root of a perfect square is an integer.

45. a. P = perimeter = 1050 ft

$$P = 4x + 3y$$
$$1050 = 4x + 3y$$
$$1050 - 4x = 3y$$
$$\frac{1050 - 4x}{3} = \frac{3y}{3}$$
$$350 - \frac{4}{3}x = y$$

b. A = area = 15,000 ft^2

$$y = 350 - \frac{4}{3}x$$
$$A = xy$$
$$15{,}000 = x\left(350 - \frac{4}{3}x\right)$$
$$15{,}000 = 350x - \frac{4}{3}x^2$$
$$\frac{4}{3}x^2 - 350x + 15{,}000 = 0$$
$$a = \frac{4}{3}, b = -350, c = 15{,}000$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-350) \pm \sqrt{(-350)^2 - 4\left(\frac{4}{3}\right)(15{,}000)}}{2\left(\frac{4}{3}\right)}$$
$$= \frac{350 \pm \sqrt{42{,}500}}{\frac{8}{3}}$$
$$= \frac{1050 \pm 3\sqrt{42{,}500}}{8}$$

The solutions are

$$x = \frac{1050 + 3\sqrt{42{,}500}}{8} \approx 208.6 \text{ and}$$
$$x = \frac{1050 - 3\sqrt{42{,}500}}{8} \approx 53.9.$$

So, the possible lengths are about 208.6 feet and about 53.9 feet.

The possible widths are

$$y = 350 - \frac{4}{3}(208.6) \approx 71.9 \text{ feet and}$$
$$y = 350 - \frac{4}{3}(53.9) \approx 278.1 \text{ feet.}$$

Chapter 9

46. $y = 15, a = -16, c = 5.5$

$$y = ax^2 + bx + c$$
$$15 = -16x^2 + bx + 5.5$$
$$\left(\frac{b}{2}\right)^2 = -16(5.5 - 15)$$
$$\left(\frac{b}{2}\right)^2 = -16(-9.5)$$
$$\left(\frac{b}{2}\right)^2 = 152$$
$$\frac{b}{2} = \pm\sqrt{152}$$
$$b = \pm 2\sqrt{152}$$

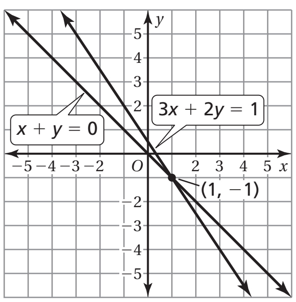
So, $b \approx 24.7$ and $b \approx -24.7$. Use the positive solution.

The minimum upward velocity is about 24.7 feet per second.

Fair Game Review

47. $x + y = 0$

$3x + 2y = 1$



The solution is $(1, -1)$.

Check:

$(1, -1)$: $x + y = 0$ $\qquad 3x + 2y = 1$

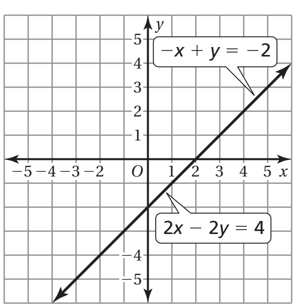
$1 + (-1) \stackrel{?}{=} 0 \qquad 3(1) + 2(-1) \stackrel{?}{=} 1$

$1 - 1 \stackrel{?}{=} 0 \qquad 3 - 2 \stackrel{?}{=} 1$

$0 = 0$ ✓ $\qquad 1 = 1$ ✓

48. $2x - 2y = 4$

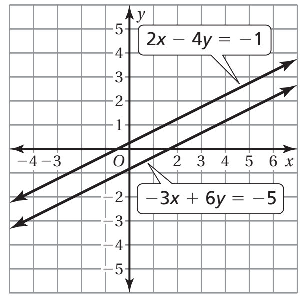
$-x + y = -2$



Both equations are the same line, so there are infinitely many solutions.

49. $2x - 4y = -1$

$-3x + 6y = -5$



The lines are parallel because they have the same slope and different y-intercepts. So, the system of linear equations has no solution.

50. A; $7x + 3x = 5x - 10$

$$10x = 5x - 10$$
$$\underline{-5x} \quad \underline{-5x}$$
$$5x = -10$$
$$\frac{5x}{5} = -\frac{10}{5}$$
$$x = -2$$

Extension 9.4

Practice (p. 485)

1. Method 1: Solve by completing the square.

$$x^2 + 14x = -8$$
$$\frac{b}{2} = \frac{14}{2} = 7$$
$$7^2 = 49$$
$$x^2 + 14x + 49 = -8 + 49$$
$$(x + 7)^2 = 41$$
$$x + 7 = \pm\sqrt{41}$$
$$x = -7 \pm \sqrt{41}$$

So, the solutions are $x = -7 + \sqrt{41} \approx -0.597$ and $x = -7 - \sqrt{41} \approx -13.403$.

Method 2: Solve using the quadratic formula.

$$x^2 + 14x = -8$$
$$x^2 + 14x + 8 = 0$$
$$a = 1, b = 14, c = 8$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-14 \pm \sqrt{14^2 - 4(1)(8)}}{2(1)} \\ &= \frac{-14 \pm \sqrt{164}}{2} \\ &= \frac{-14 \pm 2\sqrt{41}}{2} \\ &= -7 \pm \sqrt{41} \end{aligned}$$

So, the solutions are $x = -7 + \sqrt{41} \approx -0.597$ and $x = -7 - \sqrt{41} \approx -13.403$.

2. Method 1: Solve by factoring.

$$x^2 - 10x + 9 = 0$$
$$(x - 9)(x - 1) = 0$$
$$x - 9 = 0 \quad or \quad x - 1 = 0$$
$$x = 9 \quad or \quad x = 1$$

The solutions are $x = 9$ and $x = 1$.

Method 2: Solve by completing the square.

$$x^2 - 10x + 9 = 0$$
$$x^2 - 10x = -9$$
$$\frac{b}{2} = \frac{-10}{2} = -5$$
$$(-5)^2 = 25$$
$$x^2 - 10x + 25 = -9 + 25$$
$$(x - 5)^2 = 16$$
$$x - 5 = \pm 4$$
$$x = 5 \pm 4$$

So, the solutions are $x = 5 + 4 = 9$ and $x = 5 - 4 = 1$.

3. Method 1: Solve by using square roots.

$$-4x^2 + 144 = 0$$
$$-4x^2 = -144$$
$$x^2 = 36$$
$$x = \pm\sqrt{36}$$
$$x = \pm 6$$

The solutions are $x = 6$ and $x = -6$.

Method 2: Solve by factoring.

$$-4x^2 + 144 = 0$$
$$-4(x^2 - 36) = 0$$
$$x^2 - 36 = 0$$
$$x^2 = 36$$
$$x = \pm\sqrt{36}$$
$$x = \pm 6$$

The solutions are $x = 6$ and $x = -6$.

4. The equation is easily factorable, so solve by factoring.

$$x^2 + 11x - 12 = 0$$
$$(x + 12)(x - 1) = 0$$
$$x + 12 = 0 \quad or \quad x - 1 = 0$$
$$x = -12 \quad or \quad x = 1$$

The solutions are $x = -12$ and $x = 1$.

5. The equation can be written in the form $x^2 = d$, so solve using square roots.

$$9x^2 - 5 = 4$$
$$9x^2 = 9$$
$$x^2 = 1$$
$$x = \pm 1$$

The solutions are $x = 1$ and $x = -1$.

6. The coefficient of x^2 is not 1 and the equation is not easily factorable, so solve using the quadratic formula.

$$5x^2 - x - 1 = 0$$
$$a = 5, b = -1, c = -1$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-1) \pm \sqrt{1^2 - 4(5)(-1)}}{2(5)} \\ &= \frac{1 \pm \sqrt{21}}{10} \end{aligned}$$

So, the solutions are $x = \frac{1 + \sqrt{21}}{10} \approx 0.558$ and $x = \frac{1 - \sqrt{21}}{10} \approx -0.358$.

Chapter 9

7. The equation is easily factorable, so solve by factoring.

$$x^2 - 3x - 40 = 0$$
$$(x - 8)(x + 5) = 0$$
$$x - 8 = 0 \quad or \quad x + 5 = 0$$
$$x = 8 \quad or \quad x = -5$$

The solutions are $x = 8$ and $x = -5$.

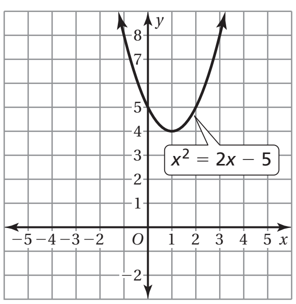
8. The coefficient of the x^2-term is 1 and the coefficient of the x-term is even, so solve by completing the square.

$$x^2 + 12x + 5 = -15$$
$$x^2 + 12x = -20$$
$$\frac{b}{2} = \frac{12}{2} = 6$$
$$6^2 = 36$$
$$x^2 + 12x + 36 = -20 + 36$$
$$(x + 6)^2 = 16$$
$$x + 6 = \pm 4$$
$$x = -6 \pm 4$$

So, the solutions are $x = -6 + 4 = -2$ and $x = -6 - 4 = -10$.

9. The equation is not factorable and the numbers are somewhat small. So, solve by graphing.

$$x^2 = 2x - 5$$



The graph does not intersect the x-axis, so there are no real solutions.

10. The equation can be written in the form $x^2 = d$. So, solve using square roots.

$$-8x^2 - 2 = 14$$
$$-8x^2 = 16$$
$$x^2 = -2$$

There are no real solutions.

11. The equation is easily factorable. So, solve by factoring.

$$x^2 + x - 12 = 0$$
$$(x + 4)(x - 3) = 0$$
$$x + 4 = 0 \quad or \quad x - 3 = 0$$
$$x = -4 \quad or \quad x = 3$$

The solutions are $x = 3$ and $x = -4$.

12. The equation is a perfect square trinomial. So, solve by factoring.

$$x^2 + 6x + 9 = 16$$
$$(x + 3)^2 = 16$$
$$x + 3 = \pm 4$$
$$x = -3 \pm 4$$

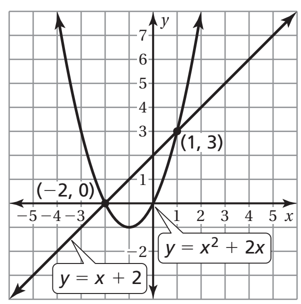
So, the solutions are $x = -3 + 4 = 1$ and $x = -3 - 4 = -7$.

Section 9.5

9.5 Activity (pp. 486–487)

1. a. Solve by graphing.

$$y = x + 2$$
$$y = x^2 + 2x$$



The solutions are $(-2, 0)$ and $(1, 3)$.

b. Solve by substitution.

$$\begin{aligned} x^2 + 2x &= x + 2 \\ \underline{-x} \quad & \quad \underline{-x} \\ x^2 + x &= 2 \\ \underline{-2} \quad & \quad \underline{-2} \\ x^2 + x - 2 &= 0 \end{aligned}$$

$(x + 2)(x - 1) = 0$

$x + 2 = 0$ *or* $x - 1 = 0$

$x = -2$ *or* $x = 1$

$x = -2$:

$y = x + 2 = -2 + 2 = 0$

$x = 1$:

$y = x + 2 = 1 + 2 = 3$

So, the solutions are $(-2, 0)$ and $(1, 3)$.

c. Solve by elimination.

$$\begin{aligned} y &= x^2 + 2x \\ \underline{-y} &\underline{= \quad -x - 2} \\ 0 &= x^2 + x - 2 \\ 0 &= (x + 2)(x - 1) \end{aligned}$$

$x + 2 = 0$ *or* $x - 1 = 0$

$x = -2$ *or* $x = 1$

$x = -2$:

$y = x + 2 = -2 + 2 = 0$

$x = 1$:

$y = x + 2 = 1 + 2 = 3$

So, the solutions are $(-2, 0)$ and $(1, 3)$.

2. a. A; $y = x^2 - 4$

$y = -x - 2$

The linear equation has a y-intercept at -2 and a slope of -1.

The quadratic equation has a y-intercept at -4 and opens up.

$$\begin{aligned} x^2 - 4 &= -x - 2 \\ \underline{+x} \quad & \quad \underline{+x} \\ x^2 + x - 4 &= -2 \\ \underline{+2} \quad & \quad \underline{+2} \\ x^2 + x - 2 &= 0 \end{aligned}$$

$(x + 2)(x - 1) = 0$

$x + 2 = 0$ *or* $x - 1 = 0$

$x = -2$ *or* $x = 1$

$x = -2$:

$y = -x - 2 = (-2) - 2 = 2 - 2 = 0$

$x = 1$:

$y = -x - 2 = -1 - 2 = -3$

So, the solutions are $(-2, 0)$ and $(1, -3)$.

b. C; $y = x^2 - 2x + 2$

$y = 2x - 2$

The linear equation has a y-intercept at -2 and a slope of 2.

The quadratic equation has a y-intercept at 2 and opens up.

$$\begin{aligned} x^2 - 2x + 2 &= 2x - 2 \\ \underline{-2x} \quad & \quad \underline{-2x} \\ x^2 - 4x + 2 &= -2 \\ \underline{+2} \quad & \quad \underline{+2} \\ x^2 - 4x + 4 &= 0 \\ (x - 2)^2 &= 0 \\ x - 2 &= 0 \\ x &= 2 \end{aligned}$$

$x = 2$:

$y = 2x - 2 = 2(2) - 2 = 4 - 2 = 2$

The solution is $(2, 2)$.

c. B; $y = x^2 + 1$

$y = x - 1$

The linear equation has a y-intercept at -1 and a slope of 1.

The quadratic equation has a y-intercept at 1 and opens up.

The graph of the quadratic equation and the graph of the linear equation do not intersect. So, there is no solution to the system.

d. D; $y = x^2 - x - 6$

$y = 2x - 2$

The linear equation has a y-intercept at -2 and a slope of 2.

The quadratic equation has a y-intercept at -6 and opens up.

$$\begin{aligned} x^2 - x - 6 &= 2x - 2 \\ \underline{-2x} \quad &\quad \underline{-2x} \\ x^2 - 3x + 6 &= -2 \\ \underline{+2} \quad &\quad \underline{+2} \\ x^2 - 3x - 4 &= 0 \end{aligned}$$

$(x - 4)(x + 1) = 0$

$x - 4 = 0$ *or* $x + 1 = 0$

$x = 4$ *or* $x = -1$

$x = 4$:

$y = 2x - 2 = 2(4) - 2 = 8 - 2 = 6$

$x = -1$:

$y = 2x - 2 = 2(-1) - 2 = -2 - 2 = -4$

So, the solutions are $(4, 6)$ and $(-1, -4)$.

3. You can solve a system of two equations when one is linear and the other is a quadratic by either graphing the system, solving by substitution, or solving by elimination.

4. *Sample answer*: Solving a system of two equations when one is linear and the other is quadratic is best done by elimination because some systems have non-integer solutions and are not factorable.

5. a. no solutions

Sample answer:

Linear equation: $y = -x - 7$

Quadratic equation: $y = x^2 + 4$

b. one solution

Sample answer:

Linear Equation: $y = x + 10$

Quadratic Equation: $y = -x^2 + 10$

c. two solutions

Sample answer:

Linear Equation: $y = \frac{1}{2}x - 1$

Quadratic Equation: $y = x^2 - x - 5$

9.5 On Your Own (pp. 488–489)

1. $y = x^2 + 9$

$y = 9$

$$\begin{aligned} y &= x^2 + 9 \\ 9 &= x^2 + 9 \\ \underline{-9} \quad &\quad \underline{-9} \\ 0 &= x^2 \\ 0 &= x \end{aligned}$$

$x = 0$:

$y = x^2 + 9 = 0^2 + 9 = 0 + 9 = 9$

The solution is $(0, 9)$.

Check: $(0, 9)$

$y = x^2 + 9$

$9 \stackrel{?}{=} 0^2 + 9$

$9 \stackrel{?}{=} 0 + 9$

$9 = 9 \checkmark$

$y = 9$

$9 = 9 \checkmark$

2. $y = -5x$

$y = x^2 - 3x - 3$

$$\begin{aligned} y &= -5x \\ x^2 - 3x - 3 &= -5x \\ \underline{+5x} \quad &\quad \underline{+5x} \\ x^2 + 2x - 3 &= 0 \end{aligned}$$

$(x + 3)(x - 1) = 0$

$x + 3 = 0$ *or* $x - 1 = 0$

$x = -3$ *or* $x = 1$

$x = -3$:

$y = x^2 - 3x - 3$

$y = (-3)^2 - 3(-3) - 3 = 9 + 9 - 3 = 15$

$x = 1$:

$y = x^2 - 3x - 3 = 1^2 - 3(1) - 3 = 1 - 3 - 3 = -5$

So, the solutions are $(-3, 15)$ and $(1, -5)$.

Check:

$(-3, 15)$:

$$\begin{aligned} y &= -5x \\ 15 &\stackrel{?}{=} -5(-3) \\ 15 &= 15 \checkmark \\ y &= x^2 - 3x - 3 \\ 15 &\stackrel{?}{=} (-3)^2 - 3(-3) - 3 \\ 15 &\stackrel{?}{=} 9 + 9 - 3 \\ 15 &= 15 \checkmark \end{aligned}$$

$(1, -5)$:

$$\begin{aligned} y &= -5x \\ -5 &\stackrel{?}{=} -5(1) \\ -5 &= 5 \checkmark \\ y &= x^2 - 3x - 3 \\ -5 &\stackrel{?}{=} 1^2 - 3(1) - 3 \\ -5 &\stackrel{?}{=} 1 - 3 - 3 \\ -5 &= -5 \checkmark \end{aligned}$$

3. $y = -3x^2 + 2x + 1$

$y = 5 - 3x$

$$\begin{aligned} y &= -3x^2 + 2x + 1 \\ 5 - 3x &= -3x^2 + 2x + 1 \\ \underline{+\,3x} & \qquad \underline{+\,3x} \\ 5 &= -3x^2 + 5x + 1 \\ \underline{-\,5} & \qquad \underline{-\,5} \\ 0 &= -3x^2 + 5x - 4 \end{aligned}$$

$a = -3, b = 5, c = -4$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{5 \pm \sqrt{5^2 - 4(-3)(-4)}}{2(-3)} \\ &= \frac{5 \pm \sqrt{-23}}{-6} \end{aligned}$$

The system has no real solutions.

4. $y = x^2 + x$

$y = x + 5$

$$\begin{aligned} y &= x^2 + x \\ -y &= \quad -x - 5 \\ \hline 0 &= x^2 - 5 \\ 5 &= x^2 \\ \pm\sqrt{5} &= x \end{aligned}$$

$x = \sqrt{5}$ *or* $x = -\sqrt{5}$

$x = \sqrt{5}$:

$y = x + 5 = \sqrt{5} + 5$

$x = -\sqrt{5}$:

$y = x + 5 = -\sqrt{5} + 5 = 5 - \sqrt{5}$

So, the solutions are $\left(\sqrt{5}, 5 + \sqrt{5}\right)$ and $\left(-\sqrt{5}, 5 - \sqrt{5}\right)$.

Check: $\left(\sqrt{5}, 5 + \sqrt{5}\right)$

$$\begin{aligned} y &= x^2 + x \\ 5 + \sqrt{5} &\stackrel{?}{=} \left(\sqrt{5}\right)^2 + \sqrt{5} \\ 5 + \sqrt{5} &= 5 + \sqrt{5} \checkmark \\ y &= x + 5 \\ 5 + \sqrt{5} &= \sqrt{5} + 5 \checkmark \end{aligned}$$

$\left(-\sqrt{5}, 5 - \sqrt{5}\right)$

$$\begin{aligned} y &= x^2 + x \\ 5 - \sqrt{5} &\stackrel{?}{=} \left(-\sqrt{5}\right)^2 + \left(-\sqrt{5}\right) \\ 5 - \sqrt{5} &= 5 - \sqrt{5} \checkmark \\ y &= x + 5 \\ 5 - \sqrt{5} &\stackrel{?}{=} -\sqrt{5} + 5 \\ 5 - \sqrt{5} &= 5 - \sqrt{5} \checkmark \end{aligned}$$

5. $y = 9x^2 + 8x - 6$

$y = 5x - 4$

$$\begin{aligned} y &= 9x^2 + 8x - 6 \\ -y &= \quad -5x + 4 \\ \hline 0 &= 9x^2 + 3x - 2 \end{aligned}$$

$a = 9, b = 3, c = -2$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-3 \pm \sqrt{3^2 - 4(9)(-2)}}{2(9)} \\ &= \frac{-3 \pm \sqrt{81}}{18} \\ &= \frac{-3 \pm 9}{18} \\ &= \frac{-1 \pm 3}{6} \end{aligned}$$

$x = \frac{-1 + 3}{6} = \frac{1}{3}$ *or* $x = \frac{-1 - 3}{6} = -\frac{2}{3}$

$x = \frac{1}{3}$:

$y = 5x - 4 = 5\left(\frac{1}{3}\right) - 4 = -\frac{7}{3}$

$x = -\frac{2}{3}$:

$y = 5x - 4 = 5\left(-\frac{2}{3}\right) - 4 = -\frac{22}{3}$

So, the solutions are $\left(\frac{1}{3}, -\frac{7}{3}\right)$ and $\left(-\frac{2}{3}, -\frac{22}{3}\right)$.

Check:

$\left(\frac{1}{3}, -\frac{7}{3}\right)$:

$$y = 9x^2 + 8x - 6$$
$$-\frac{7}{3} \stackrel{?}{=} 9\left(\frac{1}{3}\right)^2 + 8\left(\frac{1}{3}\right) - 6$$
$$-\frac{7}{3} \stackrel{?}{=} 9\left(\frac{1}{9}\right) + \frac{8}{3} - 6$$
$$-\frac{7}{3} \stackrel{?}{=} 1 + \frac{8}{3} - 6$$
$$-\frac{7}{3} = -\frac{7}{3} \checkmark$$

$$y = 5x - 4$$
$$-\frac{7}{3} \stackrel{?}{=} 5\left(\frac{1}{3}\right) - 4$$
$$-\frac{7}{3} \stackrel{?}{=} \frac{5}{3} - 4$$
$$-\frac{7}{3} = -\frac{7}{3} \checkmark$$

$\left(-\frac{2}{3}, -\frac{22}{3}\right)$:

$$y = 9x^2 + 8x - 6$$
$$-\frac{22}{3} \stackrel{?}{=} 9\left(-\frac{2}{3}\right)^2 + 8\left(-\frac{2}{3}\right) - 6$$
$$-\frac{22}{3} \stackrel{?}{=} 9\left(\frac{4}{9}\right) - \frac{16}{3} - 6$$
$$-\frac{22}{3} \stackrel{?}{=} 4 - \frac{16}{3} - 6$$
$$-\frac{22}{3} = -\frac{22}{3} \checkmark$$

$$y = 5x - 4$$
$$-\frac{22}{3} \stackrel{?}{=} 5\left(-\frac{2}{3}\right) - 4$$
$$-\frac{22}{3} \stackrel{?}{=} \frac{10}{3} - 4$$
$$-\frac{22}{3} = -\frac{22}{3} \checkmark$$

6. $y = 2x + 5$

$y = -3x^2 + x - 4$

$$\begin{array}{rl} y = & 2x + 5 \\ -y = & 3x^2 - x + 4 \\ \hline 0 = & 3x^2 + x + 9 \end{array}$$

$a = 3, b = 1, c = 9$

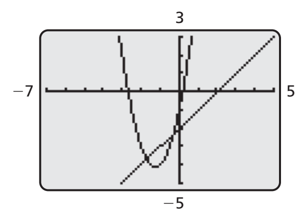
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1^2 - 4(3)(9)}}{2(3)}$$
$$= \frac{-1 \pm \sqrt{-107}}{6}$$

The system has no real solutions.

7. No; $y = 2x^2 + 5x - 1$

$y = x - 2$

Use a graphing calculator to graph the system.



The graphs of $y = 2x^2 + 5x - 1$ and $y = x - 2$ intersect at about $(-1.7, -3.7)$ and about $(-0.3, -2.3)$.

The system has two solutions.

9.5 Exercises (pp. 490–491)

Vocabulary and Concept Check

1. A solution of a system of linear and quadratic equations is a point of intersection between the linear equation and the quadratic equation.

2. Solving a system of linear and quadratic equations is similar to solving a system of linear equations because you can solve either type by elimination, substitution, or graphing. Solving a system of linear and quadratic equations is different because it involves finding the intersection(s) of a line and a parabola or solving a quadratic equation, while solving a linear system involves finding the intersection(s) of two lines or solving a linear equation.

Chapter 9

Practice and Problem Solving

3. B; $y = x^2 - 2x + 1$

$y = x + 1$

The linear equation has a y-intercept at 1 and a slope of 1. The quadratic equation has a y-intercept at 1 and opens up.

$$\begin{aligned} y &= x^2 - 2x + 1 \\ x + 1 &= x^2 - 2x + 1 \\ \underline{-x} \quad & \quad \underline{-x} \\ 1 &= x^2 - 3x + 1 \\ \underline{-1} \quad & \quad \underline{-1} \\ 0 &= x^2 - 3x \\ 0 &= x(x - 3) \end{aligned}$$

$x = 0 \ \textit{or} \ x - 3 = 0$

$x = 0 \ \textit{or} \quad x = 3$

$x = 0$: $y = x + 1 = 0 + 1 = 1$

$x = 3$: $y = x + 1 = 3 + 1 = 4$

So, the solutions are $(0, 1)$ and $(3, 4)$.

4. C; $y = x^2 + 3x + 2$

$y = -x - 3$

The linear equation has a y-intercept at -3 and a slope of -1.

The quadratic equation has a y-intercept at 2 and opens up.
The system has no real solutions.

5. A; $y = x - 1$

$y = -x^2 + x - 1$

The linear equation has a y-intercept at -1 and a slope of 1.

The quadratic equation has a y-intercept at -1 and opens down.

$$\begin{aligned} y &= \quad x - 1 \\ -x^2 + x - 1 &= \quad x - 1 \\ \underline{-x} \quad & \quad \underline{-x} \\ -x^2 - 1 &= -1 \\ \underline{+1} \quad & \quad \underline{+1} \\ -x^2 &= \quad 0 \\ x^2 &= 0 \\ x &= 0 \end{aligned}$$

$x = 0$:

$y = -x^2 + x - 1 = -0^2 + 0 - 1 = -0 + 0 - 1 = -1$

The solution is $(0, -1)$.

6. $y = x - 5$

$y = x^2 + 4x - 5$

$$\begin{aligned} y &= \quad x - 5 \\ x^2 + 4x - 5 &= \quad x - 5 \\ \underline{-x} \quad & \quad \underline{-x} \\ x^2 + 3x - 5 &= -5 \\ \underline{+5} \quad & \quad \underline{+5} \\ x^2 + 3x &= \quad 0 \\ x(x + 3) &= 0 \end{aligned}$$

$x = 0 \ \textit{or} \ x + 3 = 0$

$x = 0 \ \textit{or} \quad x = -3$

$x = 0$:

$y = x^2 + 4x - 5 = 0^2 + 4(0) - 5 = 0 + 0 - 5 = -5$

$x = -3$:

$$\begin{aligned} y &= x^2 + 4x - 5 \\ &= (-3)^2 + 4(-3) - 5 \\ &= 9 - 12 - 5 \\ &= -8 \end{aligned}$$

So, the solutions are $(0, -5)$ and $(-3, -8)$.

Check:

$(0, -5)$:
$$\begin{aligned} y &= x - 5 \\ -5 &\stackrel{?}{=} 0 - 5 \\ -5 &= -5 \checkmark \end{aligned}$$

$$\begin{aligned} y &= x^2 + 4x - 5 \\ -5 &\stackrel{?}{=} 0^2 + 4(0) - 5 \\ -5 &\stackrel{?}{=} 0 + 0 - 5 \\ -5 &= -5 \checkmark \end{aligned}$$

$(-3, -8)$:
$$\begin{aligned} y &= x - 5 \\ -8 &\stackrel{?}{=} -3 - 5 \\ -8 &= -8 \checkmark \end{aligned}$$

$$\begin{aligned} y &= x^2 + 4x - 5 \\ -8 &\stackrel{?}{=} (-3)^2 + 4(-3) - 5 \\ -8 &\stackrel{?}{=} 9 - 12 - 5 \\ -8 &= -8 \checkmark \end{aligned}$$

7. $y = -2x^2$

$y = 4x + 2$

$$\begin{aligned} y &= -2x^2 \\ 4x + 2 &= -2x^2 \\ \underline{+2x^2} &\quad \underline{+2x^2} \\ 2x^2 + 4x + 2 &= 0 \end{aligned}$$

$a = 2, b = 4, c = 2$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-4 \pm \sqrt{4^2 - 4(2)(2)}}{2(2)} \\ &= \frac{-4 \pm \sqrt{0}}{4} \\ &= \frac{-4}{4} \\ &= -1 \end{aligned}$$

$x = -1$:

$y = 4x + 2 = 4(-1) + 2 = -4 + 2 = -2$

The solution is $(-1, -2)$.

Check: $(-1, -2)$

$$\begin{aligned} y &= -2x^2 & y &= 4x + 2 \\ -2 &\stackrel{?}{=} -2(-1)^2 & -2 &\stackrel{?}{=} 4(-1) + 2 \\ -2 &\stackrel{?}{=} -2(1) & -2 &\stackrel{?}{=} -4 + 2 \\ -2 &= -2 \checkmark & -2 &= -2 \checkmark \end{aligned}$$

8. $y = -x + 7$

$y = -x^2 - 2x - 1$

$$\begin{aligned} y &= -x + 7 \\ -x^2 - 2x - 1 &= -x + 7 \\ \underline{+x} &\quad \underline{+x} \\ -x^2 - x - 1 &= 7 \\ \underline{-7} &\quad \underline{-7} \\ -x^2 - x - 8 &= 0 \\ x^2 + x + 8 &= 0 \end{aligned}$$

$a = 1, b = 1, c = 8$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{1^2 - 4(1)(8)}}{2(1)} \\ &= \frac{-1 \pm \sqrt{-31}}{2} \end{aligned}$$

The system has no real solutions.

9. $y = -x^2 + 7$

$y - 2x = 4$

$$\begin{aligned} y - 2x &= 4 \\ \underline{+2x} &\quad \underline{+2x} \\ y &= 2x + 4 \end{aligned}$$

$$\begin{aligned} y &= -x^2 + 7 \\ 2x + 4 &= -x^2 + 7 \\ \underline{+x^2} &\quad \underline{+x^2} \\ x^2 + 2x + 4 &= 7 \\ \underline{-7} &\quad \underline{-7} \\ x^2 + 2x - 3 &= 0 \end{aligned}$$

$(x + 3)(x - 1) = 0$

$x + 3 = 0$ *or* $x - 1 = 0$

$x = -3$ *or* $x = 1$

$$\begin{aligned} &x = -3: & &x = 1: \\ y - 2x &= 4 & y - 2x &= 4 \\ y - 2(-3) &= 4 & y - 2(1) &= 4 \\ y + 6 &= 4 & y - 2 &= 4 \\ y &= -2 & y &= 6 \end{aligned}$$

The solutions are $(-3, -2)$ and $(1, 6)$.

Check:

$$\begin{aligned} &(-3, -2): & &(1, 6): \\ y &= -x^2 + 7 & y &= -x^2 + 7 \\ -2 &\stackrel{?}{=} -(-3)^2 + 7 & 6 &\stackrel{?}{=} -1^2 + 7 \\ -2 &\stackrel{?}{=} -9 + 7 & 6 &\stackrel{?}{=} -1 + 7 \\ -2 &= -2 \checkmark & 6 &= 6 \checkmark \\ y - 2x &= 4 & y - 2x &= 4 \\ -2 - 2(-3) &\stackrel{?}{=} 4 & 6 - 2(1) &\stackrel{?}{=} 4 \\ -2 + 6 &\stackrel{?}{=} 4 & 6 - 2 &\stackrel{?}{=} 4 \\ 4 &= 4 \checkmark & 4 &= 4 \checkmark \end{aligned}$$

10. $y - 5 = -x^2$

$y = 5$

$$\begin{aligned} y - 5 &= -x^2 \\ \underline{+5} \quad & \quad \underline{+5} \\ y &= -x^2 + 5 \\ 5 &= -x^2 + 5 \\ \underline{-5} \quad & \quad \quad \underline{-5} \\ 0 &= -x^2 \\ 0 &= x^2 \\ 0 &= x \end{aligned}$$

$$\begin{aligned} x = 0: \quad y - 5 &= -x^2 \\ y - 5 &= -0^2 \\ y - 5 &= 0 \\ y &= 5 \end{aligned}$$

The solution is $(0, 5)$.

Check: $(0, 5)$

$$\begin{aligned} y - 5 &= -x^2 \\ 5 - 5 &\stackrel{?}{=} -0^2 \\ 0 &= 0 \checkmark \end{aligned} \qquad \begin{aligned} y &= 5 \\ 5 &= 5 \checkmark \end{aligned}$$

11. $y = 2x^2 + 3x - 4$

$y - 4x = 2$

$$\begin{aligned} y - 4x &= 2 \\ \underline{+4x} \quad & \quad \underline{+4x} \\ y &= 4x + 2 \end{aligned}$$

$$\begin{aligned} y &= 2x^2 + 3x - 4 \\ 4x + 2 &= 2x^2 + 3x - 4 \\ \underline{-4x} \quad & \quad \quad \underline{-4x} \\ 2 &= 2x^2 - x - 4 \\ \underline{-2} \quad & \quad \quad \underline{-2} \\ 0 &= 2x^2 - x - 6 \end{aligned}$$

$a = 2, b = -1, c = -6$

$$\begin{aligned} x &= \frac{-b\sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-6)}}{2(2)} \\ &= \frac{1 \pm \sqrt{49}}{4} \\ &= \frac{1 \pm 7}{4} \end{aligned}$$

$x = \frac{1 + 7}{4} = 2$ *or* $x = \frac{1 - 7}{4} = -\frac{3}{2}$

$$\begin{aligned} x = 2: \quad y - 4x &= 2 \\ y - 4(2) &= 2 \\ y - 8 &= 2 \\ y &= 10 \end{aligned}$$

$$\begin{aligned} x = -\frac{3}{2}: \quad y - 4x &= 2 \\ y - 4\left(-\frac{3}{2}\right) &= 2 \\ y + 6 &= 2 \\ y &= -4 \end{aligned}$$

So, the solutions are $(2, 10)$ and $\left(-\frac{3}{2}, -4\right)$.

Check:

$(2, 10)$:

$$\begin{aligned} y &= 2x^2 + 3x - 4 \\ 10 &\stackrel{?}{=} 2(2)^2 + 3(2) - 4 \\ 10 &\stackrel{?}{=} 2(4) + 6 - 4 \\ 10 &\stackrel{?}{=} 8 + 6 - 4 \\ 10 &= 10 \checkmark \end{aligned} \qquad \begin{aligned} y - 4x &= 2 \\ 10 - 4(2) &\stackrel{?}{=} 2 \\ 10 - 8 &\stackrel{?}{=} 2 \\ 2 &= 2 \checkmark \end{aligned}$$

$\left(-\frac{3}{2}, -4\right)$:

$$\begin{aligned} y &= 2x^2 + 3x - 4 \\ -4 &\stackrel{?}{=} 2\left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) - 4 \\ -4 &\stackrel{?}{=} 2\left(\frac{9}{4}\right) - \frac{9}{2} - 4 \\ -4 &\stackrel{?}{=} \frac{9}{2} - \frac{9}{4} - 4 \\ -4 &= -4 \checkmark \end{aligned} \qquad \begin{aligned} y - 4x &= 2 \\ -4 - 4\left(-\frac{3}{2}\right) &\stackrel{?}{=} 2 \\ -4 + 6 &\stackrel{?}{=} 2 \\ 2 &= 2 \checkmark \end{aligned}$$

12. $y = -x^2 - 2x + 2$

$y = 4x + 2$

$$\begin{array}{rcl} y &=& -x^2 - 2x + 2 \\ -\,y &=& \quad\;\; -4x - 2 \\ \hline 0 &=& -x^2 - 6x \\ 0 &=& x^2 + 6x \\ 0 &=& x(x + 6) \end{array}$$

$x = 0$ *or* $x + 6 = 0$

$x = 0$ *or* $x = -6$

$x = 0$: $y = 4x + 2 = 4(0) + 2 = 0 + 2 = 2$

$x = -6$: $y = 4x + 2 = 4(-6) + 2 = -24 + 2 = -22$

The solutions are $(0, 2)$ and $(-6, -22)$.

Check:

$(0, 2)$:

$y = -x^2 - 2x + 2$

$2 \stackrel{?}{=} -0^2 - 2(0) + 2$

$2 \stackrel{?}{=} 0 + 0 + 2$

$2 = 2 \checkmark$

$y = 4x + 2$

$2 \stackrel{?}{=} 4(0) + 2$

$2 \stackrel{?}{=} 0 + 2$

$2 = 2 \checkmark$

$(-6, -22)$:

$y = -x^2 - 2x + 2$

$-22 \stackrel{?}{=} -(-6)^2 - 2(-6) + 2$

$-22 \stackrel{?}{=} -36 + 12 + 2$

$-22 = -22 \checkmark$

$y = 4x + 2$

$-22 \stackrel{?}{=} 4(-6) + 2$

$-22 \stackrel{?}{=} -24 + 2$

$-22 = -22 \checkmark$

13. $y = -2x^2 + x - 3$

$y = 2x - 2$

$$\begin{array}{rcl} y &=& -2x^2 + x - 3 \\ -\,y &=& \quad\;\; -2x + 2 \\ \hline 0 &=& -2x^2 - x - 1 \end{array}$$

$a = -2, b = -1, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(-2)(-1)}}{2(-2)} = \frac{1 \pm \sqrt{-7}}{-4}$$

The system has no real solutions.

14. $y = 2x - 1$

$y = x^2$

$$\begin{array}{rcl} y &=& \quad\;\; 2x - 1 \\ -\,y &=& -x^2 \\ \hline 0 &=& -x^2 + 2x - 1 \\ 0 &=& x^2 - 2x + 1 \\ 0 &=& (x - 1)^2 \\ 0 &=& x - 1 \\ 1 &=& x \end{array}$$

$x = 1$: $y = x^2 = 1^2 = 1$

The solution is $(1, 1)$.

Check: $(1, 1)$

$y = 2x - 1$

$1 \stackrel{?}{=} 2(1) - 1$

$1 \stackrel{?}{=} 2 - 1$

$1 = 1 \checkmark$

$y = x^2$

$1 \stackrel{?}{=} 1^2$

$1 = 1 \checkmark$

15. $y = -2x$

$y - x^2 = 3x$

$$\begin{array}{rcl} y - x^2 &=& 3x \\ \underline{+x^2} && \underline{+x^2} \\ y &=& x^2 + 3x \end{array}$$

$$\begin{array}{rcl} y &=& \quad\;\; -2x \\ -\,y &=& -x^2 - 3x \\ \hline 0 &=& -x^2 - 5x \\ 0 &=& -x(x + 5) \end{array}$$

$-x = 0$ *or* $x + 5 = 0$

$x = 0$ *or* $x = -5$

$x = 0$:

$y - x^2 = 3x$

$y - 0^2 = 3(0)$

$y - 0 = 0$

$y = 0$

$x = -5$:

$y - x^2 = 3x$

$y - (-5)^2 = 3(-5)$

$y - 25 = -15$

$y = 10$

The solutions are $(0, 0)$ and $(-5, 10)$.

Check:

$(0, 0)$:

$$\begin{aligned} y &= -2x \\ 0 &\stackrel{?}{=} -2(0) \\ 0 &= 0 \checkmark \end{aligned}$$

$$\begin{aligned} y - x^2 &= 3x \\ 0 - 0^2 &\stackrel{?}{=} 3(0) \\ 0 - 0 &\stackrel{?}{=} 0 \\ 0 &= 0 \checkmark \end{aligned}$$

$(-5, 10)$:

$$\begin{aligned} y &= -2x \\ 10 &\stackrel{?}{=} -2(-5) \\ 10 &= 10 \checkmark \end{aligned}$$

$$\begin{aligned} y - x^2 &= 3x \\ 10 - (-5)^2 &\stackrel{?}{=} 3(-5) \\ 10 - 25 &\stackrel{?}{=} -15 \\ -15 &= -15 \checkmark \end{aligned}$$

16. $y - 1 = x^2 + x$

$y = -x - 2$

$$\begin{aligned} y - 1 &= x^2 + x \\ +1 &\quad +1 \\ y &= x^2 + x + 1 \end{aligned}$$

$$\begin{aligned} y &= x^2 + x + 1 \\ -y &= x + 2 \\ \hline 0 &= x^2 + 2x + 3 \\ -3 &= x^2 + 2x \end{aligned}$$

$\frac{b}{2} = \frac{2}{2} = 1$

$1^2 = 1$

$$\begin{aligned} -3 + 1 &= x^2 + 2x + 1 \\ -2 &= (x + 1)^2 \\ \pm\sqrt{-2} &= x + 1 \end{aligned}$$

The system has no real solutions.

17. $y = \frac{1}{2}x - 7$

$y + 4x = x^2 - 2$

$$\begin{aligned} y + 4x &= x^2 - 2 \\ -4x &\quad -4x \\ y &= x^2 - 4x - 2 \end{aligned}$$

$$\begin{aligned} y &= \tfrac{1}{2}x - 7 \\ -y &= -x^2 + 4x + 2 \\ \hline 0 &= -x^2 + \tfrac{9}{2}x - 5 \end{aligned}$$

$a = -1, b = \frac{9}{2}, c = -5$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-\frac{9}{2} \pm \sqrt{\left(\frac{9}{2}\right)^2 - 4(-1)(-5)}}{2(-1)} \\ &= \frac{-\frac{9}{2} \pm \sqrt{\frac{1}{4}}}{-2} \\ &= \frac{-\frac{9}{2} \pm \frac{1}{2}}{-2} \\ &= \frac{9}{4} \pm \frac{1}{4} \end{aligned}$$

$x = \frac{9}{4} + \frac{1}{4}$ *or* $x = \frac{9}{4} - \frac{1}{4}$

$x = \frac{5}{2}$ *or* $x = 2$

$x = \frac{5}{2}$:

$$\begin{aligned} y + 4x &= x^2 - 2 \\ y + 4\left(\frac{5}{2}\right) &= \left(\frac{5}{2}\right)^2 - 2 \\ y + 10 &= \frac{25}{4} - 2 \\ y + 10 &= \frac{17}{4} \\ y &= -\frac{23}{4} \end{aligned}$$

$x = 2$:

$$\begin{aligned} y + 4x &= x^2 - 2 \\ y + 4(2) &= 2^2 - 2 \\ y + 8 &= 4 - 2 \\ y + 8 &= 2 \\ y &= -6 \end{aligned}$$

So, the solutions are $\left(\frac{5}{2}, -\frac{23}{4}\right)$ and $(2, -6)$.

Check:

$\left(\frac{5}{2}, -\frac{23}{4}\right)$: $\quad y = \frac{1}{2}x - 7$

$$-\frac{23}{4} \stackrel{?}{=} \frac{1}{2}\left(\frac{5}{2}\right) - 7$$

$$-\frac{23}{4} \stackrel{?}{=} \frac{5}{4} - 7$$

$$-\frac{23}{4} = -\frac{23}{4} \checkmark$$

$$y + 4x = x^2 - 2$$

$$-\frac{23}{4} + 4\left(\frac{5}{2}\right) \stackrel{?}{=} \left(\frac{5}{2}\right)^2 - 2$$

$$-\frac{23}{4} + 10 \stackrel{?}{=} \frac{25}{4} - 2$$

$$\frac{17}{4} = \frac{17}{4} \checkmark$$

$(2, -6)$: $\quad y = \frac{1}{2}x - 7$

$$-6 \stackrel{?}{=} \frac{1}{2}(2) - 7$$

$$-6 \stackrel{?}{=} 1 - 7$$

$$-6 = -6 \checkmark$$

$$y + 4x = x^2 - 2$$

$$-6 + 4(2) \stackrel{?}{=} 2^2 - 2$$

$$-6 + 8 \stackrel{?}{=} 4 - 2$$

$$2 = 2 \checkmark$$

18. $y = -x^2 + 35x + 100$

$y = -5x + 275$

$$\begin{array}{rcl} y &=& -x^2 + 35x + 100 \\ -\ y &=& 5x - 275 \\ \hline 0 &=& -x^2 + 40x - 175 \\ 0 &=& x^2 - 40x + 175 \\ 0 &=& (x - 35)(x - 5) \end{array}$$

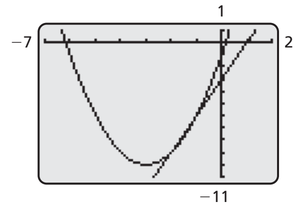
$x - 35 = 0 \quad or \quad x - 5 = 0$

$x = 35 \quad or \quad x = 5$

The attendance is the same for each movie 5 days after the movie opened and 35 days after the movie opened.

19. $y = x^2 + 6x - 1$

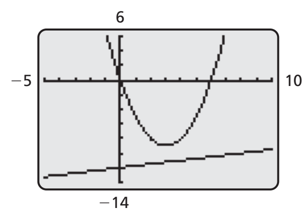
$y = 3x - 3$



The graphs of $y = x^2 + 6x - 1$ and $y = 3x - 3$ intersect at two points, $(-1, -6)$ and $(-2, -9)$.

20. $y = \frac{1}{4}x - 12$

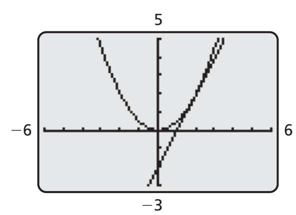
$y = x^2 - 6x$



The graphs of $y = \frac{1}{4}x - 12$ and $y = x^2 - 6x$ do not intersect, so the system has no real solutions.

21. $y = \frac{1}{2}x^2$

$y = 2x - 2$



The graphs of $y = \frac{1}{2}x^2$ and $y = 2x - 2$ intersect at only one point, $(2, 2)$.

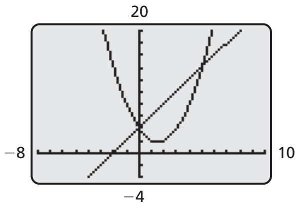
22. *Sample answer:* Solving systems of equations with a graphing calculator is usually faster, especially if the solutions do not consist of integers.

Chapter 9

23. The solver did not increase the window size on the graphing calculator to obtain the second solution.

$y = x^2 - 3x + 4$

$y = 2x + 4$



The solutions of the system of equations are $(0, 4)$ and $(5, 14)$.

24. a. $x = 1$: $y = -x^2 + 65x + 256$

$$= -1^2 + 65(1) + 256$$
$$= -1 + 65 + 256$$
$$= 320$$

$x = 34$: $y = -x^2 + 65x + 256$

$$= -34^2 + 65(34) + 256$$
$$= -1156 + 2210 + 256$$
$$= 1310$$

The graphs of the equations intersect at $(1, 320)$ and $(34, 1310)$.

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{1310 - 320}{34 - 1} = \frac{990}{33} = 30$$

$m = 30, y_1 = 320, x_1 = 1$

$$y - y_1 = m(x - x_1)$$
$$y - 320 = 30(x - 1)$$
$$y - 320 = 30x - 30$$
$$y = 30x + 290$$

The linear function that models the number of subscribers to the competitor's website is $y = 30x + 290$.

b.
$$\begin{array}{rcrr} y & = -x^2 + & 65x & + 256 \\ -\,y & = & -30x & - 290 \\ \hline 0 & = -x^2 + & 35x & - 34 \end{array}$$

$$0 = x^2 - 35x + 34$$
$$0 = (x - 34)(x - 1)$$

$x - 34 = 0$ *or* $x - 1 = 0$

$x = 34$ *or* $x = 1$

So, the system is verified because the websites have the same number of subscribers on days 1 and 34.

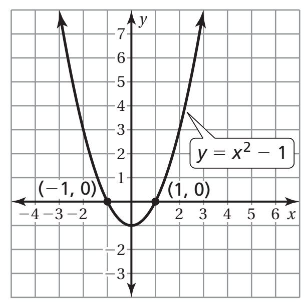
25. a. The system will have two solutions because adding 2 to c translates the line 2 units up.

b. The system will have zero real solutions because changing the linear equation to $y = c - 2$ translates the line lower and it does not intersect the graph of the quadratic function.

26. No, a system of linear and quadratic equations cannot have an infinite number of solutions because a quadratic function is never linear and must have only 0, 1, or 2 real solutions.

Fair Game Review

27. $x^2 = 1$



So, the solutions are $x = -1$ and $x = 1$.

Check:

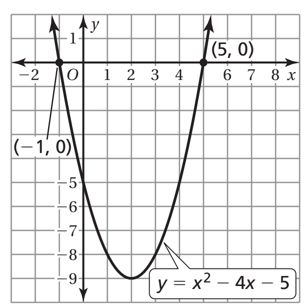
$x = -1$:

$$x^2 = 1$$
$$(-1)^2 \stackrel{?}{=} 1$$
$$1 = 1 \checkmark$$

$x = 1$:

$$x^2 = 1$$
$$1^2 \stackrel{?}{=} 1$$
$$1 = 1 \checkmark$$

28. $x^2 - 4x - 5 = 0$



So, the solutions are $x = -1$ and $x = 5$.

Check:

$x = -1$:

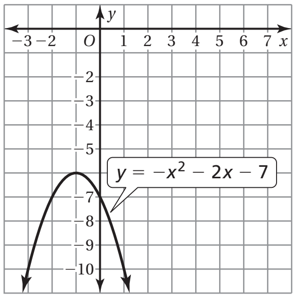
$$x^2 - 4x - 5 = 0$$
$$(-1)^2 - 4(-1) - 5 \stackrel{?}{=} 0$$
$$1 + 4 - 5 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

$x = 5$:

$$x^2 - 4x - 5 = 0$$
$$5^2 - 4(5) - 5 \stackrel{?}{=} 0$$
$$25 - 20 - 5 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

Chapter 9

29. $-x^2 = 2x + 7$



The graph of the quadratic function does not intersect the x-axis, so there are no real solutions.

30. C; $x^2 - 36 = (x + 6)(x - 6)$

Quiz 9.4–9.5 (p. 492)

1. $x^2 + 8x - 20 = 0$

$a = 1, b = 8, c = -20$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-8 \pm \sqrt{8^2 - 4(1)(-20)}}{2(1)}$$
$$= \frac{-8 \pm \sqrt{144}}{2}$$
$$= \frac{-8 \pm 12}{2}$$
$$= -4 \pm 6$$

The solutions are $x = -4 + 6 = 2$ and $x = -4 - 6 = -10$.

2.
$$\begin{aligned} 13x &= 2x^2 + 6 \\ \underline{-13x} \quad & \quad \underline{-13x} \\ 0 &= 2x^2 - 13x + 6 \end{aligned}$$

$a = 2, b = -13, c = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-13) \pm \sqrt{(-13)^2 - 4(2)(6)}}{2(2)}$$
$$= \frac{13 \pm \sqrt{121}}{4}$$
$$= \frac{13 \pm 11}{4}$$

The solutions are $x = \dfrac{13 + 11}{4} = 6$ and $x = \dfrac{13 - 11}{4} = \dfrac{1}{2}$.

3.
$$\begin{aligned} 9 - 24x &= -16x^2 \\ \underline{+16x^2} \quad & \quad \underline{+16x^2} \\ 16x^2 - 24x + 9 &= 0 \end{aligned}$$

$a = 16, b = -24, c = 9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-24) \pm \sqrt{(-24)^2 - 4(16)(9)}}{2(16)}$$
$$= \frac{24 \pm \sqrt{0}}{32}$$
$$= \frac{24 \pm 0}{32}$$
$$= \frac{24}{32}$$
$$= \frac{3}{4}$$

The only solution is $x = \dfrac{3}{4}$.

4. $x^2 + 6x - 13 = 0$

$a = 1, b = 6, c = -13$

$b^2 - 4ac = 6^2 - 4(1)(-13) = 36 + 52 = 88$

The discriminant is greater than 0, so the quadratic equation has two real solutions.

5.
$$-8x^2 - x = 5$$
$$-8x^2 - x - 5 = 0$$

$a = -8, b = -1, c = -5$

$b^2 - 4ac = (-1)^2 - 4(-8)(-5) = 1 - 160 = -159$

The discriminant is less than 0, so the quadratic equation has no real solutions.

6.
$$\begin{aligned} \frac{3}{4}x^2 &= 3x - 3 \\ \underline{-\frac{3}{4}x^2} \quad & \quad \underline{-\frac{3}{4}x^2} \\ 0 &= -\frac{3}{4}x^2 + 3x - 3 \end{aligned}$$

$a = -\dfrac{3}{4}, b = 3, c = -3$

$b^2 - 4ac = 3^2 - 4\left(-\dfrac{3}{4}\right)(-3) = 9 - 9 = 0$

The discriminant is 0, so the quadratic equation has only one real solution.

Chapter 9

7. Method 1: Solve by factoring.

$$x^2 + 10x + 21 = 0$$
$$(x + 7)(x + 3) = 0$$
$$x + 7 = 0 \quad or \quad x + 3 = 0$$
$$x = -7 \quad or \quad x = -3$$

Method 2: Solve by completing the square.

$$x^2 + 10x + 21 = 0$$
$$x^2 + 10x = -21$$
$$\frac{b}{2} = \frac{10}{2} = 5$$
$$5^2 = 25$$
$$x^2 + 10x + 25 = -21 + 25$$
$$(x + 5)^2 = 4$$
$$x + 5 = \pm 2$$
$$x = -5 \pm 2$$

The solutions are $x = -5 + 2 = -3$ and $x = -5 - 2 = -7$.

8. The coefficient of the x^2-term is 1 and the coefficient of the x-term is even. So, solve by completing the square.

$$x^2 + 4x - 11 = 0$$
$$x^2 + 4x = 11$$
$$\frac{b}{2} = \frac{4}{2} = 2$$
$$2^2 = 4$$
$$x^2 + 4x + 4 = 11 + 4$$
$$(x + 2)^2 = 15$$
$$x + 2 = \pm\sqrt{15}$$
$$x = -2 \pm \sqrt{15}$$

So, the solutions are $x = -2 + \sqrt{15} \approx 1.873$ and $x = -2 - \sqrt{15} \approx -5.873$.

9. The quadratic equation can be written in the form $x^2 = d$, so solve by using square roots.

$$-4x^2 + 1 = 0$$
$$-4x^2 = -1$$
$$x^2 = \frac{1}{4}$$
$$x = \pm\sqrt{\frac{1}{4}}$$
$$x = \pm\frac{1}{2}$$

So, the solutions are $x = \frac{1}{2}$ and $x = -\frac{1}{2}$.

10. The coefficient of the x^2-term is 1 and the coefficient of the x-term is even. So, solve by completing the square.

$$52 = x^2 - 2x$$
$$\frac{b}{2} = \frac{-2}{2} = -1$$
$$(-1)^2 = 1$$
$$52 + 1 = x^2 - 2x + 1$$
$$53 = (x - 1)^2$$
$$\pm\sqrt{53} = x - 1$$
$$1 \pm \sqrt{53} = x$$

So, the solutions are $x = 1 + \sqrt{53} \approx 8.280$, and $x = 1 - \sqrt{53} \approx -6.280$.

11. $y = x^2 - 16$

$y = -7$

$$\begin{aligned} y &= x^2 - 16 \\ -7 &= x^2 - 16 \\ \underline{+16} & \quad \underline{+16} \\ 9 &= x^2 \\ \pm 3 &= x \end{aligned}$$

$x = 3$ and $x = -3$

$x = 3$:

$y = x^2 - 16 = 3^2 - 16 = 9 - 16 = -7$

$x = -3$:

$y = x^2 - 16 = (-3)^2 - 16 = 9 - 16 = -7$

So, the solutions are $(3, -7)$ and $(-3, -7)$.

Chapter 9

12. $y = x^2 + 2x + 1$

$y = 2x + 2$

$$\begin{aligned} y &= x^2 + 2x + 1 \\ -y &= \quad\ -2x - 2 \\ \hline 0 &= x^2 \qquad\ - 1 \\ 1 &= x^2 \\ \pm 1 &= x \end{aligned}$$

$x = 1$ and $x = -1$

$x = 1$:

$y = 2x + 2 = 2(1) + 2 = 2 + 2 = 4$

$x = -1$:

$y = 2x + 2 = 2(-1) + 2 = -2 + 2 = 0$

So, the solutions are $(1, 4)$ and $(-1, 0)$.

13. $y = x^2 - 5x + 8$

$y = -3x + 4$

$$\begin{aligned} y &= x^2 - 5x + 8 \\ -y &= \qquad\ 3x - 4 \\ \hline 0 &= x^2 - 2x + 4 \\ -4 &= x^2 - 2x \end{aligned}$$

$\frac{b}{2} = \frac{-2}{2} = -1$

$(-1)^2 = 1$

$-4 + 1 = x^2 - 2x + 1$

$-3 = (x - 1)^2$

$\sqrt{-3} = x + 1$

The system has no real solutions.

14. $y = 3t^2 + 8t + 20$ (Type 1)

$y = 27t + 60$ (Type 2)

a. As t increases. Type 1 grows more quickly because each t-value is being squared in the quadratic model.

b.
$$\begin{aligned} y &= 3t^2 + \ 8t + 20 \\ -y &= \qquad -27t - 60 \\ \hline 0 &= 3t^2 - 19t - 40 \end{aligned}$$

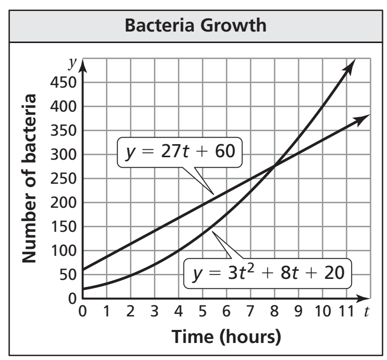
$a = 3, b = -19, c = -40$

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-19) \pm \sqrt{(-19)^2 - 4(3)(-40)}}{2(3)} \\ &= \frac{19 \pm \sqrt{841}}{6} \\ &= \frac{19 \pm 29}{6} \end{aligned}$$

$t = \frac{19 + 29}{6} = 8$ or $t = \frac{19 - 29}{6} = -\frac{5}{3}$. Use the positive solution because t represents time.

So, the numbers of Type 1 and Type 2 bacteria are the same after 8 hours.

c.



By looking at the graph, you can see there are more Type 1 bacteria after 8 hours.

There are more Type 2 bacteria from 0 to 8 hours.

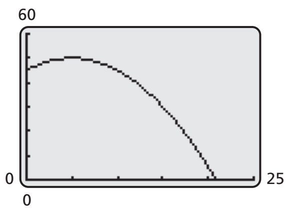
Chapter 9

15. $y = 50$

$$y = -0.2x^2 + 2x + 45$$
$$50 = -0.2x^2 + 2x + 45$$
$$5 = -0.2x^2 + 2x$$
$$-25 = x^2 - 10x$$
$$\frac{b}{2} = \frac{-10}{2} = -5$$
$$(-5)^2 = 25$$
$$-25 + 25 = x^2 - 10x + 25$$
$$0 = (x - 5)^2$$
$$0 = x - 5$$
$$5 = x$$

Because x represents the number of years after 2000, the average monthly bill for the customer's cell phone is $50 in 2005.

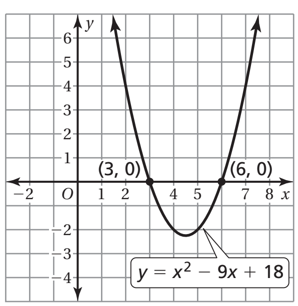
16.



No, the model cannot be used for future years because after year 2020, the customer's average monthly bill will be negative, which is impossible.

Chapter 9 Review (pp. 493–495)

1. $x^2 - 9x + 18 = 0$



So, the solutions are $x = 3$ and $x = 6$.

Check:

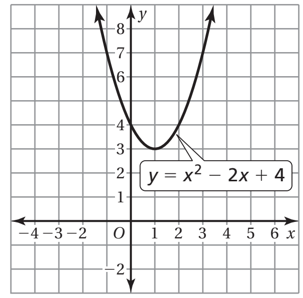
$x = 3$:

$$x^2 - 9x + 18 = 0$$
$$3^2 - 9(3) + 18 \stackrel{?}{=} 0$$
$$9 - 27 + 18 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

$x = 6$:

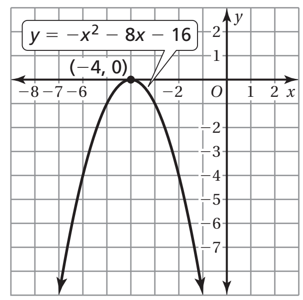
$$x^2 - 9x + 18 = 0$$
$$6^2 - 9(6) + 18 \stackrel{?}{=} 0$$
$$36 - 54 + 18 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

2. $x^2 - 2x = -4$



The graph does not intersect the x-axis, so the quadratic equation has no real solutions.

3. $-8x - 16 = x^2$



The only solution is $x = -4$.

Check: $x = -4$

$$-8x - 16 = x^2$$
$$-8(-4) - 16 \stackrel{?}{=} (-4)^2$$
$$32 - 16 \stackrel{?}{=} 16$$
$$16 = 16 \checkmark$$

4. $x^2 - 10 = -10$

$$x^2 = 0$$
$$x = 0$$

5. $4x^2 = -100$

$$x^2 = -25$$

The equation has no real solutions.

6. $(x + 2)^2 = 64$

$$x + 2 = \pm 8$$
$$x = -2 \pm 8$$

The solutions are $x = -2 + 8 = 6$ and $x = -2 - 8 = -10$.

7. $x^2 + x + 10 = 0$

$$x^2 + x = -10$$

$$\frac{b}{2} = \frac{1}{2}$$

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$x^2 + x + \frac{1}{4} = -10 + \frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 = -\frac{39}{4}$$

$$x + \frac{1}{2} = \sqrt{-\frac{39}{4}}$$

The quadratic equation has no real solutions.

8. $x^2 + 2x + 5 = 4$

$$x^2 + 2x = -1$$

$$\frac{b}{2} = \frac{2}{2} = 1$$

$$1^2 = 1$$

$$x^2 + 2x + 1 = -1 + 1$$

$$(x + 1)^2 = 0$$

$$x + 1 = 0$$

$$x = -1$$

The solution is $x = -1$.

9. $2x^2 - 4x = 10$

$$x^2 - 2x = 5$$

$$\frac{b}{2} = \frac{-2}{2} = -1$$

$$(-1)^2 = 1$$

$$x^2 - 2x + 1 = 5 + 1$$

$$(x - 1)^2 = 6$$

$$x - 1 = \pm\sqrt{6}$$

$$x = 1 \pm \sqrt{6}$$

The solutions are $x = 1 + \sqrt{6} \approx 3.449$ and $x = 1 - \sqrt{6} \approx -1.449$.

10. A = area, P = perimeter, ℓ = length, w = width

$A = 46.75 \text{ cm}^2, w = \ell - 3$

$$A = \ell w$$

$$46.75 = \ell(\ell - 3)$$

$$46.75 = \ell^2 - 3\ell$$

$$\frac{b}{2} = \frac{-3}{2}$$

$$\left(-\frac{3}{2}\right)^2 = \frac{9}{4} = 2.25$$

$$46.75 + 2.25 = \ell^2 - 3\ell + 2.25$$

$$49 = (\ell - 1.5)^2$$

$$\pm 7 = \ell - 1.5$$

$$1.5 \pm 7 = \ell$$

The solutions are $\ell = 1.5 + 7 = 8.5$ and $\ell = 1.5 - 7 = -5.5$. Use the positive solution.

The length of the credit card is 8.5 centimeters and the width is $8.5 - 3 = 5.5$ centimeters.

$\ell = 8.5, w = 5.5$

$$P = 2\ell + 2w = 2(8.5) + 2(5.5) = 17 + 11 = 28$$

So, the perimeter of the credit card is 28 centimeters.

11. $x^2 + 2x - 15 = 0$

$a = 1, b = 2, c = -15$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{2^2 - 4(1)(-15)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{64}}{2}$$

$$= \frac{-2 \pm 8}{2}$$

$$= -1 \pm 4$$

The solutions are $x = -1 + 4 = 3$ and $x = -1 - 4 = -5$.

Chapter 9

12. $2x^2 - x + 8 = 3$

$2x^2 - x + 5 = 0$

$a = 2, b = -1, c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(5)}}{2(2)}$$
$$= \frac{1 \pm \sqrt{1 - 40}}{4}$$
$$= \frac{1 \pm \sqrt{-39}}{4}$$

The quadratic equation has no real solutions.

13. $-5x^2 + 10x = 5$

$-5x^2 + 10x - 5 = 0$

$a = -5, b = 10, c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-10 \pm \sqrt{10^2 - 4(-5)(-5)}}{2(-5)}$$
$$= \frac{-10 \pm \sqrt{0}}{-10}$$
$$= \frac{-10 \pm 0}{-10}$$
$$= \frac{-10}{-10}$$
$$= 1$$

The solution is $x = 1$.

14. The quadratic equation can be written in the form $x^2 = d$, so solve using square roots.

$x^2 - 121 = 0$

$x^2 = 121$

$x = \pm 11$

The solutions are $x = 11$ and $x = -11$.

15. The quadratic equation is a perfect square trinomial so solve by factoring.

$x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0$

$x - 2 = 0$

$x = 2$

The solution is $x = 2$.

16. The coefficient of the x^2-term is 1 and the coefficient of the x-term is even. So, solve by completing the square.

$x^2 - 4x = -1$

$\frac{b}{2} = \frac{-4}{2} = -2$

$(-2)^2 = 4$

$x^2 - 4x + 4 = -1 + 4$

$(x - 2)^2 = 3$

$x - 2 = \pm\sqrt{3}$

$x = 2 \pm \sqrt{3}$

The solutions are $x = 2 + \sqrt{3} \approx 3.732$ and $2 - \sqrt{3} \approx 0.268$.

17. $y = x^2 - 2x - 4$

$y = -5$

$y = x^2 - 2x - 4$

$-5 = x^2 - 2x - 4$

$-1 = x^2 - 2x$

$\frac{b}{2} = \frac{-2}{2} = -1$

$(-1)^2 = 1$

$-1 + 1 = x^2 - 2x + 1$

$0 = (x - 1)^2$

$0 = x - 1$

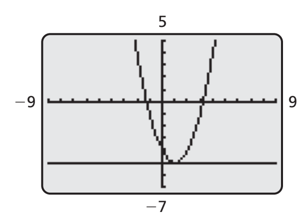
$1 = x$

$x = 1$:

$y = x^2 - 2x - 4 = 1^2 - 2(1) - 4 = 1 - 2 - 4 = -5$

So, the solution is $(1, -5)$.

Check:



18. $y = x^2 - 9$

$y = 2x + 5$

$$\begin{array}{rcl} y & = & x^2 \qquad - 9 \\ -\,y & = & \quad -2x - 5 \\ \hline 0 & = & x^2 - 2x - 14 \end{array}$$

$14 = x^2 - 2x$

$\frac{b}{2} = \frac{-2}{2} = -1$

$(-1)^2 = 1$

$14 + 1 = x^2 - 2x + 1$

$15 = (x - 1)^2$

$\pm\sqrt{15} = x - 1$

$1 \pm \sqrt{15} = x$

$x = 1 + \sqrt{15}$ and $x = 1 - \sqrt{15}$

$x = 1 + \sqrt{15}$:

$y = 2x + 5$

$y = 2\left(1 + \sqrt{15}\right) + 5$

$= 2 + 2\sqrt{15} + 5$

$= 7 + 2\sqrt{15}$

$x = 1 - \sqrt{15}$:

$y = 2x + 5$

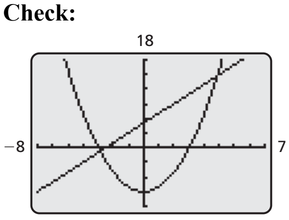
$y = 2\left(1 - \sqrt{15}\right) + 5$

$= 2 - 2\sqrt{15} + 5$

$= 7 - 2\sqrt{15}$

So, the solutions are $\left(1 + \sqrt{15}, 7 + 2\sqrt{15}\right)$ and $\left(1 - \sqrt{15}, 7 - 2\sqrt{15}\right)$. In decimal form, the solutions are about $(4.873, 14.746)$ and about $(-2.873, -0.746)$.

Check:



19. $y = 2 - 3x$

$y = -x^2 - 5x - 4$

$$\begin{array}{rcl} y & = & \qquad -3x + 2 \\ -\,y & = & x^2 + 5x + 4 \\ \hline 0 & = & x^2 + 2x + 6 \end{array}$$

$-6 = x^2 + 2x$

$\frac{b}{2} = \frac{2}{2} = 1$

$1^2 = 1$

$-6 + 1 = x^2 + 2x + 1$

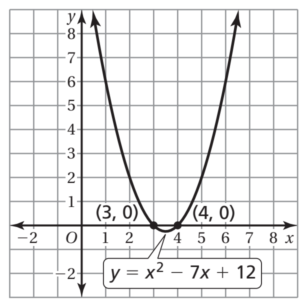
$-5 = (x + 1)^2$

$\pm\sqrt{-5} = x + 1$

The system of equations has no real solutions.

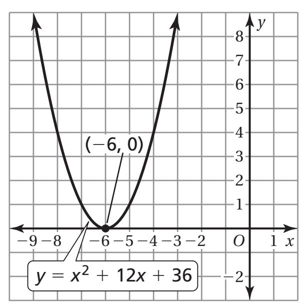
Chapter 9 Test (p. 496)

1. $x^2 - 7x + 12 = 0$



So, the solutions are $x = 3$ and $x = 4$.

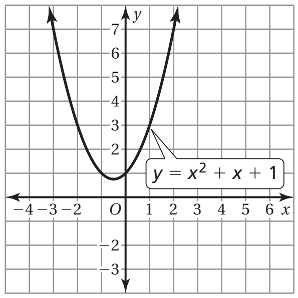
2. $x^2 + 12x = -36$



So, the solution is $x = -6$.

Chapter 9

3. $x + 1 = -x^2$



The graph does not intersect the x-axis. So, the quadratic equation has no real solutions.

4.
$$14 = 2x^2$$
$$7 = x^2$$
$$\pm\sqrt{7} = x$$

The solutions are $x = \sqrt{7} \approx 2.646$ and $x = -\sqrt{7} \approx -2.646$.

5.
$$x^2 + 9 = 5$$
$$x^2 = -4$$
$$x = \pm\sqrt{-4}$$

The quadratic equation has no real solutions.

6.
$$(4x + 3)^2 = 16$$
$$4x + 3 = \pm 4$$
$$4x = -3 \pm 4$$
$$x = \frac{-3 \pm 4}{4}$$

The solutions are $x = \frac{-3 + 4}{4} = \frac{1}{4}$ and $x = \frac{-3 - 4}{4} = -\frac{7}{4}$.

7.
$$x^2 - 8x + 15 = 0$$
$$x^2 - 8x = -15$$
$$\frac{b}{2} = \frac{-8}{2} = -4$$
$$(-4)^2 = 16$$
$$x^2 - 8x + 16 = -15 + 16$$
$$(x - 4)^2 = 1$$
$$x - 4 = \pm 1$$
$$x = 4 \pm 1$$

The solutions are $x = 4 + 1 = 5$ and $x = 4 - 1 = 3$.

8.
$$x^2 - 6x = 10$$
$$\frac{b}{2} = \frac{-6}{2} = -3$$
$$(-3)^2 = 9$$
$$x^2 - 6x + 9 = 10 + 9$$
$$(x - 3)^2 = 19$$
$$x - 3 = \pm\sqrt{19}$$
$$x = 3 \pm \sqrt{19}$$

The solutions are $x = 3 + \sqrt{19} \approx 7.359$ and $x = 3 - \sqrt{19} \approx -1.359$.

9.
$$x^2 - 8x = -9$$
$$\frac{b}{2} = \frac{-8}{2} = -4$$
$$(-4)^2 = 16$$
$$x^2 - 8x + 16 = -9 + 16$$
$$(x - 4)^2 = 7$$
$$x - 4 = \pm\sqrt{7}$$
$$x = 4 \pm \sqrt{7}$$

The solutions are $x = 4 + \sqrt{7} \approx 6.646$ and $x = 4 - \sqrt{7} \approx 1.354$.

10.
$$16 = x^2 - 16x - 20$$
$$36 = x^2 - 16x$$
$$\frac{b}{2} = \frac{-16}{2} = -8$$
$$(-8)^2 = 64$$
$$36 + 64 = x^2 - 16x + 64$$
$$100 = (x - 8)^2$$
$$\pm 10 = x - 8$$
$$8 \pm 10 = x$$

The solutions are $x = 8 + 10 = 18$ and $x = 8 - 10 = -2$.

11. $5x^2 + x - 4 = 0$

$a = 5, b = 1, c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1^2 - 4(5)(-4)}}{2(5)}$$
$$= \frac{-1 \pm \sqrt{81}}{10}$$
$$= \frac{-1 \pm 9}{10}$$

The solutions are $x = \frac{-1 + 9}{10} = \frac{4}{5}$ and $x = \frac{-1 - 9}{10} = -1$.

12. $9x^2 + 6x + 1 = 0$

$a = 9, b = 6, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-6 \pm \sqrt{6^2 - 4(9)(1)}}{2(9)}$$
$$= \frac{-6 \pm \sqrt{0}}{18}$$
$$= \frac{-6 \pm 0}{18}$$
$$= \frac{-6}{18}$$
$$= -\frac{1}{3}$$

The solutions is $x = -\frac{1}{3}$.

13. $-2x^2 + 3x + 7 = 0$

$a = -2, b = 3, c = 7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{3^2 - 4(-2)(7)}}{2(-2)}$$
$$= \frac{-3 \pm \sqrt{65}}{-4}$$

The solutions are $x = \frac{-3 + \sqrt{65}}{-4} \approx -1.266$ and $x = \frac{-3 - \sqrt{65}}{-4} \approx -2.766$.

14. $y = 4x^2 - 4x + 1$

$a = 4, b = -4, c = 1$

$b^2 - 4ac = (-4)^2 - 4(4)(1) = 16 - 16 = 0$

The discriminant is 0, so the graph of $y = 4x^2 - 4x + 1$ intersects the x-axis at one point.

15. The quadratic equation is easily factorable, so solve by factoring.

$x^2 - 9x - 10 = 0$

$(x - 10)(x + 1) = 0$

$x - 10 = 0$ *or* $x + 1 = 0$

$x = 10$ *or* $x = -1$

The solutions are $x = 10$ and $x = -1$.

16. $y = x^2 - 4x - 2$

$y = -4x + 2$

$$\begin{array}{rcl} y &=& x^2 - 4x - 2 \\ -\ y &=& 4x - 2 \\ \hline 0 &=& x^2 - 4 \end{array}$$

$4 = x^2$

$\pm 2 = x$

$x = 2$ and $x = -2$

$x = 2$:

$y = -4x + 2 = -4(2) + 2 = -8 + 2 = -6$

$x = -2$:

$y = -4x + 2 = -4(-2) + 2 = 8 + 2 = 10$

So, the solutions are $(2, -6)$ and $(-2, 10)$.

17. $y = -5x^2 + x - 1$

$y = -7$

$y = -5x^2 + x - 1$

$-7 = -5x^2 + x - 1$

$0 = -5x^2 + x + 6$

$a = -5, b = 1, c = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1^2 - 4(-5)(6)}}{2(-5)}$$
$$= \frac{-1 \pm \sqrt{121}}{-10}$$
$$= \frac{-1 \pm 11}{-10}$$

$x = \frac{-1 + 11}{-10} = -1$ *or* $x = \frac{-1 - 11}{-10} = \frac{6}{5}$

$x = -1$:

$$\begin{aligned} y &= -5x^2 + x - 1 \\ &= -5(-1)^2 + (-1) - 1 \\ &= -5(1) - 1 - 1 \\ &= -5 - 1 - 1 \\ &= -7 \end{aligned}$$

$x = \dfrac{6}{5}$:

$$\begin{aligned} y &= -5x^2 + x - 1 \\ &= -5\left(\frac{6}{5}\right)^2 + \left(\frac{6}{5}\right) - 1 \\ &= -5\left(\frac{36}{25}\right) + \frac{6}{5} - 1 \\ &= \frac{-36}{5} + \frac{6}{5} - 1 \\ &= -7 \end{aligned}$$

So, the solutions are $(-1, -7)$ and $\left(\dfrac{6}{5}, -7\right)$.

18. A = area, b = base, h = height

$A = 35 \text{ ft}^2, b = x + 4, h = x$

$$A = \frac{1}{2}bh$$

$$35 = \left(\frac{1}{2}\right)(x + 4)(x)$$

$$35 = \frac{1}{2}x^2 + 2x$$

$$70 = x^2 + 4x$$

$$\frac{b}{2} = \frac{4}{2} = 2$$

$$2^2 = 4$$

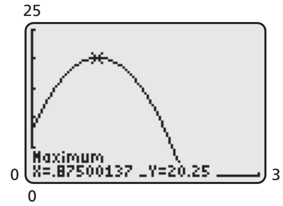
$$\begin{aligned} 70 + 4 &= x^2 + 4x + 4 \\ 74 &= (x + 2)^2 \\ \pm\sqrt{74} &= x + 2 \\ -2 \pm \sqrt{74} &= x \end{aligned}$$

$x = -2 + \sqrt{74} \approx 6.6$ and

$x = -2 - \sqrt{74} \approx -10.6$. Use the positive solution.

The height of the triangle is 6.6 feet, so the length of the base is $6.6 + 4 = 10.6$ feet.

19. $h = -16t^2 + 28t + 8$



Maximum height is
20.25 feet $\times$ 1 point per foot $= 20.25$ points.

Time in air is
2 seconds $\times$ 5 feet per second $= 10$ points.

Perfect landing is 25 points $= 25$ points.

So, the snowboarder earns
$20.25 + 10 + 25 = 55.25$ points.

Ch. 9 Standardized Test Practice (pp. 497–499)

1. B; A = area, s = side length $= x - 6$

$$A = s^2 = (x - 6)^2 = x^2 - 12x + 36$$

2. I; F. $(x - 25)^2 = 5$

$$\begin{aligned} x - 25 &= \pm\sqrt{5} \\ x &= 25 \pm \sqrt{5} \end{aligned}$$

The solutions are $x = 25 + \sqrt{5} \approx 27.236$ and $x = 25 - \sqrt{5} \approx 22.764$.

G. $-4x^2 = 0$

$$\begin{aligned} x^2 &= 0 \\ x &= 0 \end{aligned}$$

The solution is $x = 0$.

H. $(3x + 1)^2 = 9$

$$\begin{aligned} 3x + 1 &= \pm 3 \\ 3x &= -1 \pm 3 \\ x &= \frac{-1 \pm 3}{3} \end{aligned}$$

The solutions are $x = \dfrac{-1 + 3}{3} = \dfrac{2}{3}$ and $x = \dfrac{-1 - 3}{3} = -\dfrac{4}{3}$.

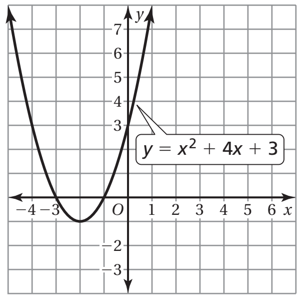
I. $2x^2 + 1 = -1$

$$\begin{aligned} 2x^2 &= -2 \\ x^2 &= -1 \\ x &= \pm\sqrt{-1} \end{aligned}$$

The quadratic equation $2x^2 + 1 = -1$ has no real solutions.

Chapter 9

3. D; $y = x^2 + 4x + 3$



By looking at the graph of $y = x^2 + 4x + 3$, you can see $x = -2$ is the minimum value, not the maximum value.

4. -4; $f(x) = -12$

$$\begin{aligned} f(x) &= 5x + 8 \\ -12 &= 5x + 8 \\ -20 &= 5x \\ -4 &= x \end{aligned}$$

The value of x that makes $f(x) = -12$ is $x = -4$.

5. H; $y = -x^2 - 3x + 1$

The axis of symmetry goes through the vertex (in this case, the maximum) of the parabola. Because the vertex (or maximum) is at $\left(-\frac{3}{2}, \frac{13}{4}\right)$, the axis of symmetry is $x = -\frac{3}{2}$.

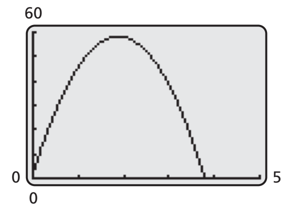
6. C; $3x^2 + x - 1 = 0$

$a = 3, b = 1, c = -1$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{1^2 - 4(3)(-1)}}{2(3)} \\ &= \frac{-1 \pm \sqrt{13}}{6} \end{aligned}$$

The exact solutions are $x = \frac{-1 + \sqrt{13}}{6}$ and $x = \frac{-1 - \sqrt{13}}{6}$.

7. I; $h = -16t^2 + 60t + 2$



By looking at the graph, you can see that the ball is kicked from a height of 2 feet. All the other statements are false.

8. B; $y = x^2 + 2x - 8$
$y = 5x + 2$

$$\begin{array}{rcl} y &=& x^2 + 2x - 8 \\ -\ y &=& - 5x - 2 \\ \hline 0 &=& x^2 - 3x - 10 \end{array}$$

$a = 1, b = -3, c = -10$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)} \\ &= \frac{3 \pm \sqrt{49}}{2} \\ &= \frac{3 \pm 7}{2} \end{aligned}$$

$x = \frac{3 + 7}{2} = 5$ and $x = \frac{3 - 7}{2} = -2$

$x = 5$:

$y = 5x + 2 = 5(5) + 2 = 25 + 2 = 27$

$x = -2$:

$y = 5x + 2 = 5(-2) + 2 = -10 + 2 = -8$

So, the system has two solutions, $(5, 27)$ and $(-2, -8)$.

9. F; The graphs in answers H and I are decreasing at some points, while graph G is a linear function. Only graph F is increasing exponentially, showing exponential growth.

Chapter 9

10. $y = 3x^2 + 3x + 4$

Part A:

The coefficient of the x^2-term is positive, so the graph opens up.

Part B:

The constant term is 4, so the y-intercept is $(0, 4)$.

Part C:

$y = 3x^2 + 3x + 4$

$y = 3(x^2 + x) + 4$

$\frac{b}{2} = \frac{1}{2}, \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$y = 3\left(x + \frac{1}{2}\right)^2 + \frac{13}{4}$

The axis of symmetry is $x = -\frac{1}{2}$.

Part D:

$y = 3\left(x + \frac{1}{2}\right)^2 + \frac{13}{4}$

The vertex is $\left(-\frac{1}{2}, \frac{13}{4}\right)$.

11. A; Jamie added 49 to the left side of the equation but not to the right.

12. 114;

$1.5x^2 - 6x = 13$

$1.5x^2 - 6x - 13 = 0$

$a = 1.5, b = -6, c = -13$

$b^2 - 4ac = (-6)^2 - 4(1.5)(-13) = 36 + 78 = 114$

The value of the discriminant is 114.

13. G; The range is $y \le 9$.

The zeros are $x = -1$ and $x = 5$.

The maximum occurs at the vertex because the function opens down.

The domain is all real numbers because the graph continues infinitely in both directions.